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**NO 897 / MAY 2008**

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**DSGE-MODELLING**

**WHEN AGENTS  
ARE IMPERFECTLY  
INFORMED**

by Paul De Grauwe





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EUROSYSTEM



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# DSGE-MODELLING WHEN AGENTS ARE IMPERFECTLY INFORMED

by Paul De Grauwe<sup>2</sup>



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## Abstract

DSGE-models have become important tools of analysis not only in academia but increasingly in the board rooms of central banks. The success of these models has much to do with the coherence of the intellectual framework it provides. The limitations of these models come from the fact that they make very strong assumptions about the cognitive abilities of agents in understanding the underlying model. In this paper we relax this strong assumption. We develop a stylized DSGE-model in which individuals use simple rules of thumb (heuristics) to forecast the future inflation and output gap. We compare this model with the rational expectations version of the same underlying model. We find that the dynamics predicted by the heuristic model differs from the rational expectations version in some important respects, in particular in their capacity to produce endogenous economic cycles.

Keyword: DSGE-model, imperfect information, heuristics, animal spirits

JEL Classification: E10, E32, D 83

### *Non-technical summary*

The most fundamental development in macroeconomics during the last few decades has been the systematic incorporation of the paradigm of the utility maximizing forward looking and fully informed agent into macroeconomic models. One of the most successful implementations of this development is to be found in the Dynamic Stochastic General Equilibrium models (DSGE-models) that are increasingly used in central banks for policy analysis. There can be no doubt that this approach to macroeconomics has important advantages compared to the previous macroeconomic modeling approaches. The main advantage is that it provides for a coherent and self-contained framework of analysis.

There are also problems in this new modeling approach. The most important one is the informational assumption underlying the DSGE-models. This is that agents are assumed to understand the structure of the underlying model. This follows directly from the rational expectations assumption which requires agents to use the underlying model structure to make forecasts.

The scientific evidence from other sciences (psychology, brain sciences) casts doubts about the plausibility of this assumption. It is no exaggeration to say that there is now strong evidence that individual agents suffer from deep cognitive problems limiting their capacity to understand and to process the complexity of the information they receive. In general the cognitive problem agents face leads them to use simple rules ("heuristics") to guide their behaviour. They do this not because they are irrational, but rather because the complexity of the world is overwhelming. In a way it can be said that using heuristics is a rational response of agents who are aware of their limited capacity to understand the world.

Thus, while DSGE-models have led to important new insights thanks to the coherence of their intellectual framework, there is a need to go beyond the overly restrictive informational assumption that requires agents to fully understand the complexity of the world in which they live. In this paper we develop an alternative macroeconomic model that incorporates the idea that agents use simple rules (heuristics) in forecasting and we contrast the results of this "heuristic model" with a stylized version of the DSGE-model. The purpose is not to show that the alternative model is better (this can only be done by empirical testing which is not done in this paper), but rather to

highlight the differences in the dynamics that arise from using different informational assumptions.

The novel feature of the model is that agents use simple rules, heuristics, to forecast the future. These rules are subjected to a selection mechanism. Put differently, agents endogenously select the forecasting rules that have delivered the highest fitness in the past. This selection mechanism acts as a disciplining device on the kind of rules that are acceptable. Since agents use different heuristics we also obtain heterogeneity.

The ensuing “heuristic model” produces a number of results that distinguishes it from the rational expectations DSGE-models. First, the heuristic model is capable of generating endogenous cycles based on waves of optimism and pessimism. This dynamics is akin to what Keynes called animal spirits. Second, in contrast to the rational expectations DSGE-models the inertia in output and prices is generated internally in the model, instead of being “imported” by assuming ad-hoc lag structures. Third, due to its non-linearity, the heuristic model produces a degree of uncertainty about the transmission of monetary policy shocks that is different from the uncertainty obtained in DSGE-models. In the latter linear models, uncertainty about the effects of monetary policy shocks arises only because of the lack of precision in the estimation of the structural parameters of the model. In the heuristic model there is an additional dimension to uncertainty. This is that the same policy shock can have different effects depending on the state of the economy, including the degree of optimism and pessimism agents have about the future. True, DSGE-models can potentially produce similar results. However, these have routinely been excluded by linearizing an otherwise non-linear model.

The research presented in this paper should be considered to be preliminary. In order to be convincing as an alternative modeling strategy, a rigorous empirical evaluation of the model will be necessary, whereby the predictions of the model are confronted with the data. In addition, the menu of heuristics which is extremely small in this paper will have to be broadened so that the selection of the “fittest” rules can occur using a wider pool of possible rules.

## 1. Introduction

The most fundamental development in macroeconomics during the last few decades has been the systematic incorporation of the paradigm of the utility maximizing forward looking and fully informed agent into macroeconomic models. This development started with the rational expectations revolution of the 1970s, which taught us that macroeconomic models can only be accepted if agents' expectations are consistent with the underlying model structure. The real business cycle theory (RBC) introduced the idea that macroeconomic models should be “micro-founded”, i.e. should be based on dynamic utility maximization. While RBC models had no place for price rigidities and other inertia, the New Keynesian School systematically introduced rigidities into similar micro-founded models. These developments occurred in the ivory towers of academia for several decades until in recent years these models were implemented empirically in such a way that they have now become tools of analysis in the boardrooms of central banks. The most successful implementation of these developments are to be found in the Dynamic Stochastic General Equilibrium models (DSGE-models) that are increasingly used in central banks for policy analysis (see Smets and Wouters(2003), Christiano, et al.(2007, Smets and Wouters(2007), Adjemian, et al. (2007)).

There can be no doubt that this approach to macroeconomics has important advantages compared to the previous macroeconomic modeling approaches. The main advantage is that it provides for a coherent and self-contained framework of analysis. This creates a great intellectual appeal. There is no need to invoke ad-hoc assumptions about how agents behave and how they make forecasts. Rational expectations and utility maximization provide the discipline about what is acceptable in modeling the behaviour of agents.

There are also problems in this new modeling approach. The most important one is the informational assumption underlying the DSGE-models. This is that agents are assumed to understand the structure of the underlying model. This follows directly from the rational expectations assumption which requires agents to use the underlying model structure to make forecasts.

The scientific evidence from other sciences (psychology, brain sciences) casts doubts about the plausibility of this assumption. It is no exaggeration to say that there is now strong evidence that individual agents suffer from deep cognitive problems limiting their capacity to understand and to process the complexity of the information they receive.

Many anomalies that challenge the rational expectations assumption were discovered (see Thaler(1994) for spirited discussions of these anomalies; see also Camerer and Lovo, (1999), Read and van Leeuwen, 1998, Della Vigna(2007)). We just mention "anchoring" effects here, whereby agents who do not fully understand the world in which they live are highly selective in the way they use information and concentrate on the information they understand or the information that is fresh in their minds. This anchoring effect explains why agents often extrapolate recent movements in prices.

In general the cognitive problem agents face leads them to use simple rules ("heuristics") to guide their behaviour (see Gabaix, Laibson, Moloche, and Weinberg, 2006). They do this not because they are irrational, but rather because the complexity of the world is overwhelming. In a way it can be said that using heuristics is a rational response of agents who are aware of their limited capacity to understand the world. The challenge when we try to model heuristics will be to introduce discipline in the selection of rules so as to avoid that "everything becomes possible".

One important implication of the assumption that agents know the underlying model's structure is that all agents are the same. They all use the same information set including the information embedded in the underlying model. As a result, DSGE-models routinely restrict the analysis to a representative agent to fully describe how all agents in the model process information. There is no heterogeneity in the use and the processing of information in these models. This reduces the usefulness of DSGE-models for the analysis of short-term and medium-term macroeconomic problems which is about the dynamics of aggregating heterogeneous behaviour and beliefs<sup>1</sup>.

Thus, while DSGE-models have led to important new insights thanks to the coherence of their intellectual framework, there is a need to go beyond the overly restrictive informational assumption that requires agents to fully understand the complexity of

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<sup>1</sup> There have been attempts to model heterogeneity of information processing in rational expectations models. These have been developed mainly in asset market models. Typically, it is assumed in these models that some agents are fully informed (rational) while others, the noise traders, are not. See e.g. De Long, et al. (1990).

the world in which they live. In this paper we develop an alternative macroeconomic model that incorporates the idea that agents use simple rules (heuristics) in forecasting and we contrast the results of this “heuristic model” with a stylized version of the DSGE-model, which will be labeled the “rational model”. The purpose is not to show that the alternative model is better (this can only be done by empirical testing which is not done in this paper), but rather to highlight the differences in the dynamics that arise from using different informational assumptions.

## **2. A heuristic model**

In this section we describe how an alternative modeling strategy can be developed. We do this by presenting a standard aggregate-demand-aggregate supply model augmented with a Taylor rule. The novel feature of the model is that agents use simple rules, heuristics, to forecast the future. These rules are subjected to a selection mechanism. Put differently, agents will endogenously select the forecasting rules that have delivered the highest fitness in the past. This selection mechanism acts as a disciplining device on the kind of rules that are acceptable. Since agents use different heuristics we also obtain heterogeneity. This, as will be shown, creates endogenous business cycles.

We will contrast the behaviour of this model with a similar model that incorporates rational expectations, and that we interpret as a stylized version of DSGE-models. This comparison will also allow us to focus on some crucial differences in the transmission of shocks, in particular of monetary policy shocks.

Obviously, the approach presented here is not the only possible one. In fact, a large literature has emerged attempting to introduce imperfect information into macroeconomic models. These attempts have been based mainly on the statistical learning approach pioneered by Sargent(1993) and Evans and Honkapohja(2001). This literature leads to important new insights (see e.g. Gaspar and Smets(2006), Orphanides and Williams(2004), Milani(2007)). However, we feel that this approach still loads individual agents with too many cognitive skills that they probably do not possess in the real world. A similar criticism can be developed against another approach at modeling imperfect information based on “rational inattention” (see Mackowiak and Wiederholt(2005), Sims(2005)).

Our approach is also not the first attempt to introduce heuristics into macroeconomic models. Recently, Brazier et al. (2006) have done so in the context of an overlapping generations model, and Branch and Evans(2006) have developed models with imperfectly informed agents. In addition, there is a large literature of behavioural finance models that now incorporate the view that agents are limited in their cognitive skills and use heuristics to guide their behaviour and forecasting (see Brock and Hommes(1997), Lux and Marchesi(2000), De Grauwe and Grimaldi(2006)).

## 2.1 The model

The model consists of an aggregate demand equation, an aggregate supply equation and a Taylor rule.

The aggregate demand equation can be derived from dynamic utility maximization. This produces an Euler equation in the same vein as in DSGE-models. We obtain

$$y_t = a_1 \tilde{E}_t y_{t+1} + (1 - a_1) y_{t-1} + a_2 (r_t - \tilde{E}_t \pi_{t+1}) + \varepsilon_t \quad (1)$$

where  $y_t$  is the output gap in period  $t$ ,  $r_t$  is the nominal interest rate,  $\pi_t$  is the rate of inflation, and  $\varepsilon_t$  is a white noise disturbance term.  $\tilde{E}_t$  is the expectations operator where the tilde above  $E$  refers to expectations that are not formed rationally. We will specify this process subsequently. We follow the procedure introduced in DSGE-models of adding a lagged output in the demand equation. This is usually justified by invoking habit formation. We keep this assumption here as we want to compare the heuristic model with the DSGE-rational expectations model. However, we will show in section 5 that we do not really need this inertia-building device to generate inertia in the endogenous variables.

The aggregate supply equation can be derived from profit maximization of individual producers. We assume as in DSGE-models a Calvo pricing rule, which leads to a lagged inflation variable in the equation<sup>2</sup>. The supply curve can also be interpreted as a New Keynesian Philips curve. We obtain:

---

<sup>2</sup> It is now standard in DSGE-models to use a pricing equation in which marginal costs enter on the right hand side. Such an equation is derived from profit maximisation in a world of imperfect competition. It allows introducing more detail into the model and makes it possible to specify

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 y_t + \eta_t \quad (2)$$

Finally the Taylor rule describes the behaviour of the central bank

$$r_t = c_1 (\pi_t - \pi_t^*) + c_2 y_t + c_3 r_{t-1} + u_t \quad (3)$$

where  $\pi_t^*$  is the inflation target which for the sake of convenience will be set equal to 0. Note that we assume, as is commonly done, that the central bank smoothens the interest rate. This smoothing behaviour is represented by the lagged interest rate in equation (3). Ideally, the Taylor rule should be formulated using a forward looking inflation variable, i.e. central banks set the interest rate on the basis of their forecasts about the rate of inflation. We have not done so here in order to maintain simplicity in the model.

We assume that agents use simple rules (heuristics) to forecast the future output and inflation. The way we proceed is as follows. We start with a very simple heuristics for forecasting and apply it to the forecasting rules of future output. We assume that because agents do not fully understand how the output gap is determined, their forecasts are biased. We assume that some agents are optimistic and systematically bias the output gap upwards, others are pessimistic and systematically bias the output gap downwards.

$$\text{The optimists are defined by } \tilde{E}_t^{opt} y_{t+1} = g \quad (4)$$

$$\text{The pessimists are defined by } \tilde{E}_t^{pes} y_{t+1} = -g \quad (5)$$

where  $g > 0$  expresses the degree of bias in estimating the output gap. We will interpret  $2g$  to express the divergence in beliefs among agents about the output gap.

The market forecast is obtained as a weighted average of these two forecasts, i.e.

$$\tilde{E}_t y_{t+1} = \alpha_{opt,t} \tilde{E}_t^{opt} y_{t+1} + \alpha_{pes,t} \tilde{E}_t^{pes} y_{t+1} \quad (6)$$

$$\tilde{E}_t y_{t+1} = \alpha_{opt,t} g - \alpha_{pes,t} g \quad (7)$$

$$\text{and } \alpha_{opt,t} + \alpha_{pes,t} = 1 \quad (8)$$

---

productivity shocks better. It also allows for analyzing how shocks in markups affect the economy. We have not tried to introduce this feature here (see Galí(), Smets and Wouters(2003)).



where  $\alpha_{opt,t}$  and  $\alpha_{pes,t}$  are the weights of optimists, receptively, pessimists in the market.

A methodological issue arises here. The forecasting rules (heuristics) introduced here are not derived at the micro level and then aggregated. Instead, they are imposed ex post, once the demand and supply equations are derived. This has also been the approach in the learning literature pioneered by Evans and Honkapohja(2001). Ideally one would like to derive the heuristics from the micro-level in an environment in which agents experience cognitive problems. Our knowledge about how to model this behaviour at the micro level<sup>3</sup> and how to aggregate it is too sketchy, however, and we have not tried to do so.

As indicated earlier, agents are rational in the sense that they continuously evaluate their forecast performance. We follow Brock and Hommes(1997) in specifying the procedure agents follow in this evaluation process. Recently, Branch and Evans(2006) introduced this selection mechanism in a macroeconomic model.

Agents compute the forecast performance of the different heuristics as follows:

$$U_{opt,t} = -\sum_{k=1}^{\infty} \omega_k [y_{t-k} - \tilde{E}_{opt,t-k-1} y_{t-k}]^2 \quad (9)$$

$$U_{pes,t} = -\sum_{k=1}^{\infty} \omega_k [y_{t-k} - \tilde{E}_{pes,t-k-1} y_{t-k}]^2 \quad (10)$$

where  $U_{opt,t}$  and  $U_{pes,t}$  are the forecast performances of the optimists and pessimists, respectively. These are defined as the mean squared forecasting errors (MSFEs) of the optimistic and pessimistic forecasting rules;  $\omega_k$  are geometrically declining weights.

The proportion of agents using the optimistic and the pessimistic forecasting rules is then determined in the following way (Brock-Hommes(1997))<sup>4</sup>:

$$\alpha_{opt,t} = \frac{\exp(\gamma U_{opt,t})}{\exp(\gamma U_{opt,t}) + \exp(\gamma U_{pes,t})} \quad (11)$$

<sup>3</sup> Psychologists and brains scientists struggle to understand how our brain processes information. There is as yet no generally accepted model we could use to model the micro-foundations of information processing.

<sup>4</sup> Such a specification is often used in discrete choice theory. See Anderson, de Palma, and Thisse, (1992)

$$\alpha_{pes,t} = \frac{\exp(\gamma U_{pes,t})}{\exp(\gamma U_{opt,t}) + \exp(\gamma U_{pes,t})} = 1 - \alpha_{opt,t} \quad (12)$$

Equation (11) says that as the past forecast performance of the optimists improves relative to that of the pessimists more agents will select the optimistic belief about the output gap for their future forecasts. As a result the proportion of agents using the optimistic rule increases. Equation (12) has a similar interpretation. The parameter  $\gamma$  measures the “intensity of choice”, i.e. the intensity with which agents allow their choice for a particular heuristic to depend on past forecast performance. In the limit when  $\gamma = \infty$  only one, the best performing heuristic, will be selected.

Note that this selection mechanism is the disciplining device introduced in this model on the kind of rules of behaviour that are acceptable. Only those rules that pass the fitness test remain in place. The others are weeded out. In contrast with the disciplining device implicit in rational expectations models which implies that agents have superior cognitive capacities, we do not have to make such an assumption here.

It is also useful to point out that the selection mechanism used here can be interpreted as an evolutionary mechanism that allows high forecasting performance to spread throughout the economy through replication.

Agents also make forecasts of inflation in this model. At this stage of the analysis we will simply assume that all agents perceive the central bank’s announced inflation target  $\pi_t^*$  to be fully credible. They use this value as their forecast of future inflation, i.e.  $\tilde{E}_t \pi_{t+1} = \pi_t^*$  (where for the sake of simplicity we assume the inflation target to be equal to 0). We will extend this simple inflation forecasting process in a later section when we will also assume that there is heterogeneity of beliefs in the inflation forecasting process. We keep homogeneity of beliefs here to focus on the impact of heterogeneity in the forecasting of future output gaps.

The solution of the model is found by first substituting (3) into (1) and rewriting in matrix notation. This yields:

$$\begin{bmatrix} 1 & -b_2 \\ -a_2 c_1 & 1 - a_2 c_2 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \begin{bmatrix} b_1 & 0 \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} \tilde{E}_t \pi_{t+1} \\ \tilde{E}_t y_{t+1} \end{bmatrix} + \begin{bmatrix} 1 - b_1 & 0 \\ 0 & 1 - a_1 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ a_2 c_3 \end{bmatrix} r_{t-1} + \begin{bmatrix} \eta_t \\ a_2 u_t + \varepsilon_t \end{bmatrix}$$

or

$$\mathbf{A} \mathbf{Z}_t = \mathbf{B} \tilde{\mathbf{E}}_t \mathbf{Z}_t + \mathbf{C} \mathbf{Z}_{t-1} + \mathbf{b} r_{t-1} + \mathbf{v}_t \quad (13)$$

where bold characters refer to matrices and vectors. The solution for  $\mathbf{Z}_t$  is given by

$$\mathbf{Z}_t = \mathbf{A}^{-1} [\mathbf{B} \tilde{\mathbf{E}}_t \mathbf{Z}_t + \mathbf{C} \mathbf{Z}_{t-1} + \mathbf{b} r_{t-1} + \mathbf{v}_t] \quad (14)$$

The solution exists if the matrix  $\mathbf{A}$  is non-singular, i.e. if  $(1-a_2c_2)a_2b_2c_1 \neq 0$ . The system (14) describes the solution for  $y_t$  and  $\pi_t$  given the forecasts of  $y_t$  and  $\pi_t$ . The latter have been specified in equations (4) to (12) and can be substituted into (14). Finally, the solution for  $r_t$  is found by substituting  $y_t$  and  $\pi_t$  obtained from (14) into (3).

Our research strategy consists in comparing the dynamics of this heuristic model with the same structural model (aggregate demand equation (1), aggregate supply equation (2) and Taylor rule equation (3)) under rational expectations which we interpret as a stylized DSGE-model. .

The model consisting of equations (1) to (3) can be written in matrix notation as follows:

$$\begin{bmatrix} 1 & -b_2 & 0 \\ 0 & 1 & -a_2 \\ -c_1 & -c_2 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ r_t \end{bmatrix} = \begin{bmatrix} b_1 & 0 & 0 \\ -a_2 & a_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_t \pi_{t+1} \\ E_t y_{t+1} \\ E_t r_{t+1} \end{bmatrix} + \begin{bmatrix} 1-b_1 & 0 & 0 \\ 0 & 1-a_1 & 0 \\ 0 & 0 & a_3 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \varepsilon_t \\ u_t \end{bmatrix}$$

$$\mathbf{\Omega} \mathbf{Z}_t = \mathbf{\Phi} E_t \mathbf{Z}_t + \mathbf{\Lambda} \mathbf{Z}_{t-1} + \mathbf{v}_t \quad (15)$$

$$\mathbf{Z}_t = \mathbf{\Omega}^{-1} [\mathbf{\Phi} E_t \mathbf{Z}_t + \mathbf{\Lambda} \mathbf{Z}_{t-1} + \mathbf{v}_t] \quad (16)$$

This model can be solved under rational expectations using the Binder-Pesaran(1996) procedure.

## 2.2 Calibrating the heuristic and the rational model

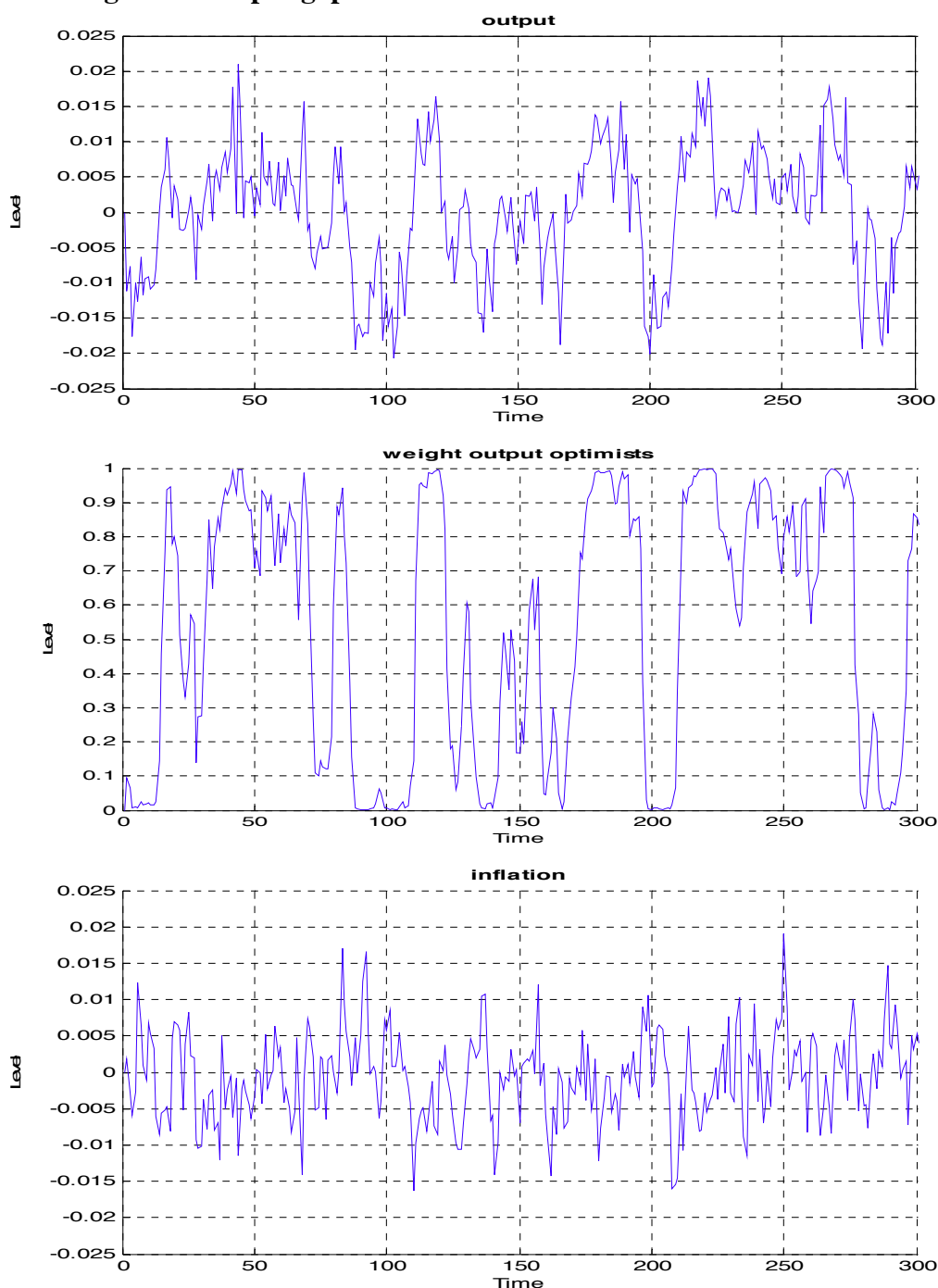
We proceed by calibrating the model. In appendix A we present the parameters used in the calibration exercise. We have calibrated the model in such a way that the time units can be considered to be months. In section 7 we present a sensitivity analysis of the main results to changes in the main parameters of the model.

We show the results of a simulation exercise in which the three shocks (demand shocks, supply shocks and interest rate shocks) are i.i.d. with standard deviations of 0.5%.

We first present a simulation in the time domain. Figure 1 shows the time pattern of output and inflation produced by the heuristic model. We observe a strong cyclical movement in the output gap. The source of these cyclical movements is seen to be the weight of optimists and pessimists in the market (see second panel of figure 1). The model in fact generates endogenous waves of optimism and pessimism. During some periods pessimists dominate and this translates into below average output growth. These pessimistic periods are followed by optimistic periods when optimistic forecasts tend to dominate and the growth rate of output is above average. These waves of optimism and pessimism are essentially unpredictable. Other realizations of the shocks produce different cycles. (In appendix B we give a few additional examples).

These endogenously generated cycles in output are reminiscent of what Keynes called “animal spirits”. In our model these animal spirits are created by a self-fulfilling mechanism that can be described as follows. A series of random shocks creates the possibility that one of the two forecasting rules, say the optimistic one, delivers a higher payoff, i.e. a lower MSFE. This attracts agents that were using the pessimistic rule. The “contagion-effect” leads to an increasing use of the optimistic belief to forecast the output-gap, which in turn stimulates aggregate demand. Optimism is therefore self-fulfilling. A boom is created. At some point, negative stochastic shocks make a dent in the MSFE of the optimistic forecasts. The pessimistic belief becomes attractive and therefore fashionable again. The economy turns around.

**Figure 1: Output gap and inflation in heuristic model**



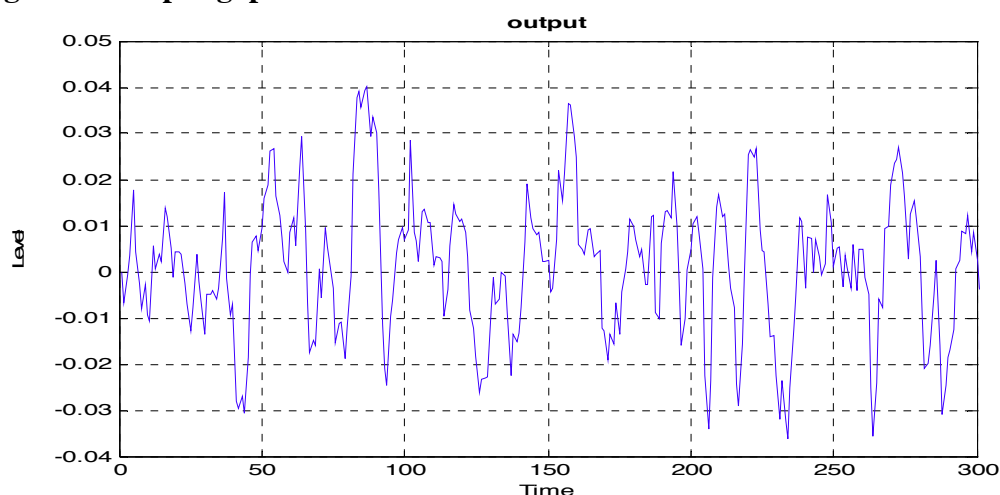
From figure 1 (third panel) we observe that inflation is relatively stable and fluctuates around the target (set at 0) in a relatively narrow band. This result has everything to do with our assumption that agents are homogeneous in giving full credibility to the inflation target of the central bank. We will return to this when we introduce heterogeneity among agents in their perception of the credibility of the central bank's inflation target.

We contrast these results with those obtained using the model under rational expectations. We use the same structural model with the same parameter values for the aggregate demand, supply and Taylor equations. In addition the shocks are the same with the same iid structure.

We show the results in figure 2. Two differences stand out. First the rational expectations model does not produce clear cyclical movements in the output gap. In a way this is not surprising: the shocks are white noise and the transmission mechanism exhibits a minimal degree of inertia. In full-fledged DSGE-models the inertia is more complex and the shocks typically exhibit autoregressive patterns that are important in producing cyclical movements in output. Thus our results suggest that the cycles produced in the DSGE models come to a large extent from outside the model. We return to this issue in section 6 where we analyze the degree of inertia produced by the two models.

Second, the volatility of output and inflation is higher in the rational expectations model compared to the heuristic model. This can also be seen from table 1 where we show the standard deviations of the output gap and inflation in the two models. Again this has to do with the minimal inertia assumed in the underlying structural model. Much of the attempt to fit the rational expectations model (DSGE-models) has consisted in adding additional lags so as to produce more persistence and less short-term volatility.

**Figure 2: Output gap and inflation in the rational model**



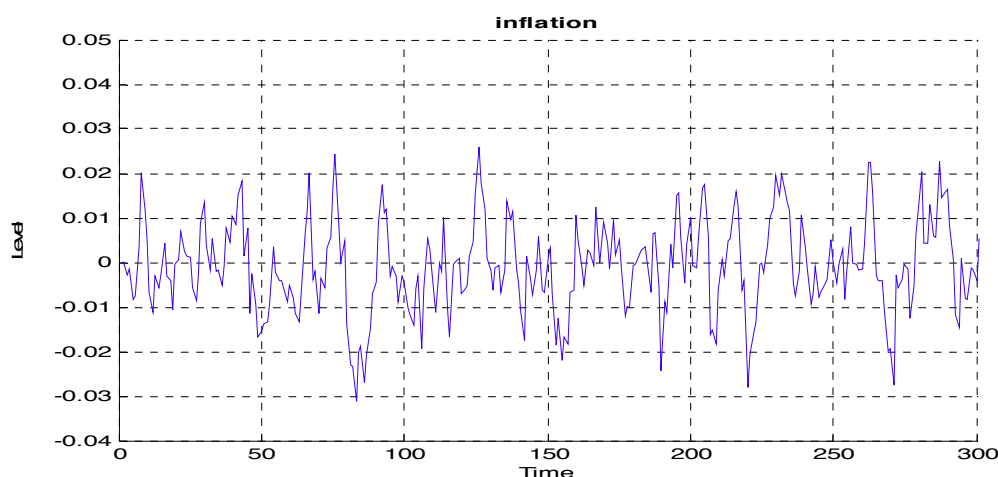


Table 1 : Standard deviations of output gap and inflation

	heuristic model	rational model
output gap	0.86	1.35
inflation	0.56	0.89

Note: these standard deviations are the averages obtained from simulating the model 1000 times, each time over 1000 periods

### 2.3 Impulse responses in the heuristic and the rational model

The next step in the analysis is to compute the impulse responses to shocks. Here we focus on the impulse responses to an interest rate shock, defined as plus one standard deviation of the shock in the Taylor equation.

The peculiarity of the heuristic model is that for the same parameters of the model the impulse responses are different for each realization of the stochastic shocks. This contrasts with the rational expectations model where the impulse response functions are not sensitive to the realization of the stochastic shocks (keeping the parameters unchanged). We will return to this difference and give it an interpretation.

Figure 3 shows the mean impulse responses to an interest rate shock. We constructed the mean response by simulating the model 100 times with 100 different realizations of the shocks. We then computed the mean response together with the standard deviations. Figure 3 shows the mean response (the dotted lines are the mean response + and - 2 standard deviations; note also that we introduced the shock after 100 periods). We obtain the standard result of an interest rate shock on output and

inflation. However, the uncertainty surrounding this result is considerable at least in the short run.

Where does this uncertainty come from? Not from parameter uncertainty. We use the same parameters in constructing all our impulse responses. The answer is that in this heuristic model each realization of the shocks creates different waves of optimism and pessimism. We could also call this “market sentiments”. Thus a shock that occurs in period 100 in one simulation happens in a different market sentiment than the same shock in another simulation. In addition, the shock itself affects market sentiments. As a result, the short-term effects of the same interest rate shock become very hard to predict. We observe from figure 3 that a significant part of the output and inflation effects are positive in the short run. In section 2.6 we elaborate further on this theme and illustrate how particular differences in “market sentiments” affect the impulse responses to shocks.

Another way to interpret this result is to say that the timing of the shock is important. The same shocks applied at different times can have very different short-term effects on inflation and output. In other words, history matters. This contrasts with what rational expectations models tell us. In these models the timing of the shock does not matter. In this sense the rational expectations model is a-historic<sup>5</sup>.

Note that the uncertainty about the impulse responses tends to disappear in the long run, as the effect of short-term differences in market sentiments disappears.

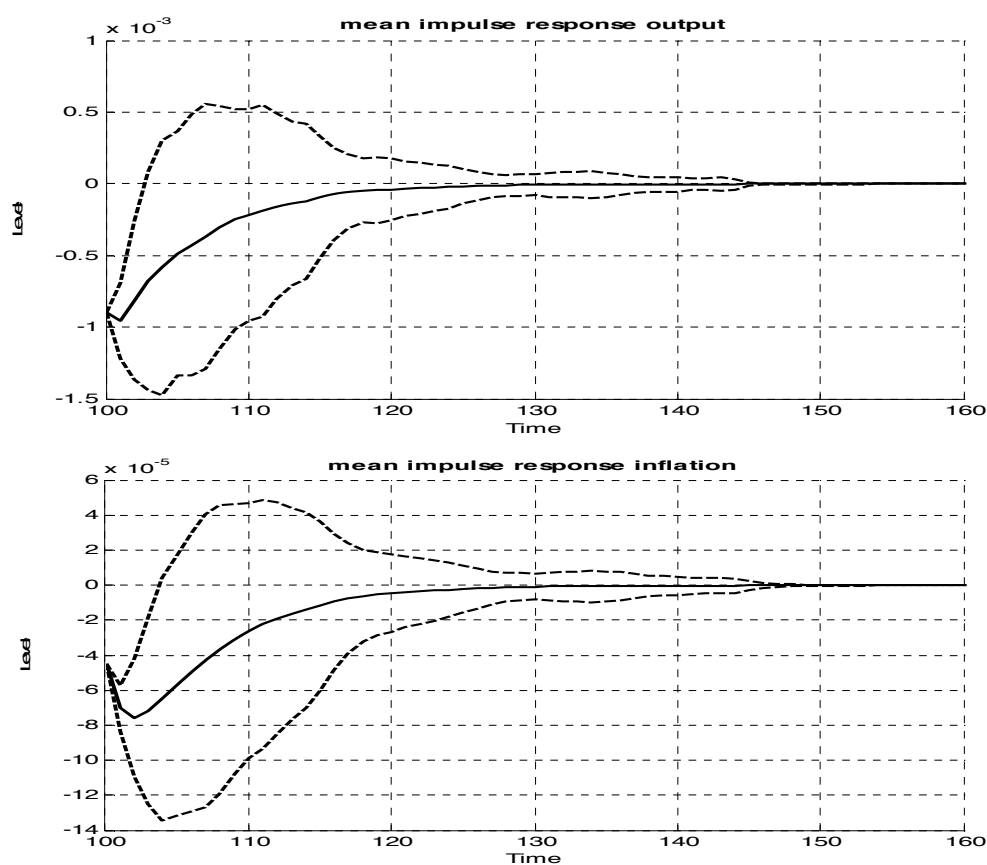
We computed similar impulse responses to an interest rate shock in the rational expectations model. The results are shown in figure 4. The first thing to note is that there is no uncertainty about these impulse response functions, as long as the parameters of the model are known with certainty. This contrasts with the heuristic model where even if we know the parameters of the model with certainty, we are still uncertain about the impulse responses because the latter depend on the realizations of the shocks (and thus market sentiments). In the rational expectations model, uncertainty about the effects of monetary policy only arises because the parameters of the model are not known with certainty.

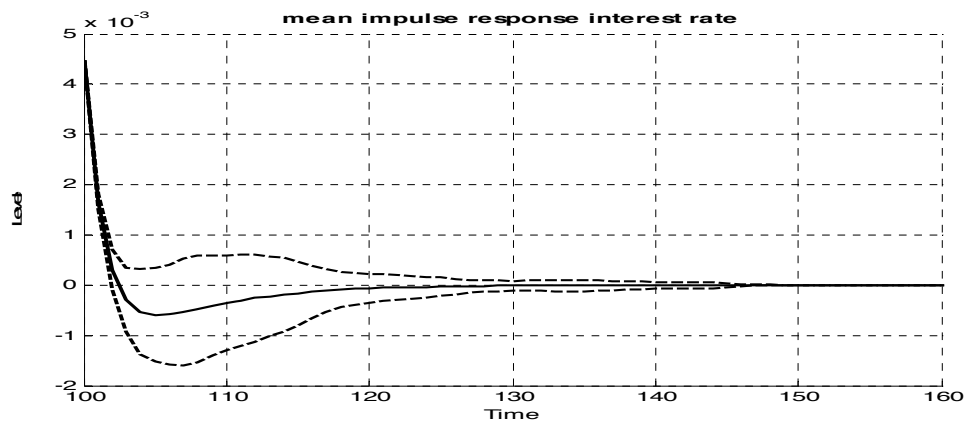
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<sup>5</sup> Michael Woodford has claimed that rational expectations models of the kind analyzed here have an element of historic dependence. This follows from the fact the existence of lags in the model. The historic dependence we are talking about here is of another nature.

This difference in the nature of uncertainty in a heuristic and a rational expectations model has everything to do with the fact that the former has non-linear features while the latter is linear. Thus the additional uncertainty produced by the heuristic model, i.e. the dependence of the impulse response functions on the state of the economy is the outcome of its non-linearity. Rational expectations models including the DSGE-models traditionally impose some linearization procedure. This is done for the sake of mathematical simplicity. It leads to a problem though. If the microfoundation of the model leads to a non-linear model, it is important to know how this non-linearity (which is part of the micro-foundation) affects the dynamics generated by the model. Eliminating these non-linearities amounts to destroying information that is relevant to predict the transmission of shocks. This may not matter much for the long run, but since the DSGE-models have the ambition of forecasting the transmission process, it is of significant importance.

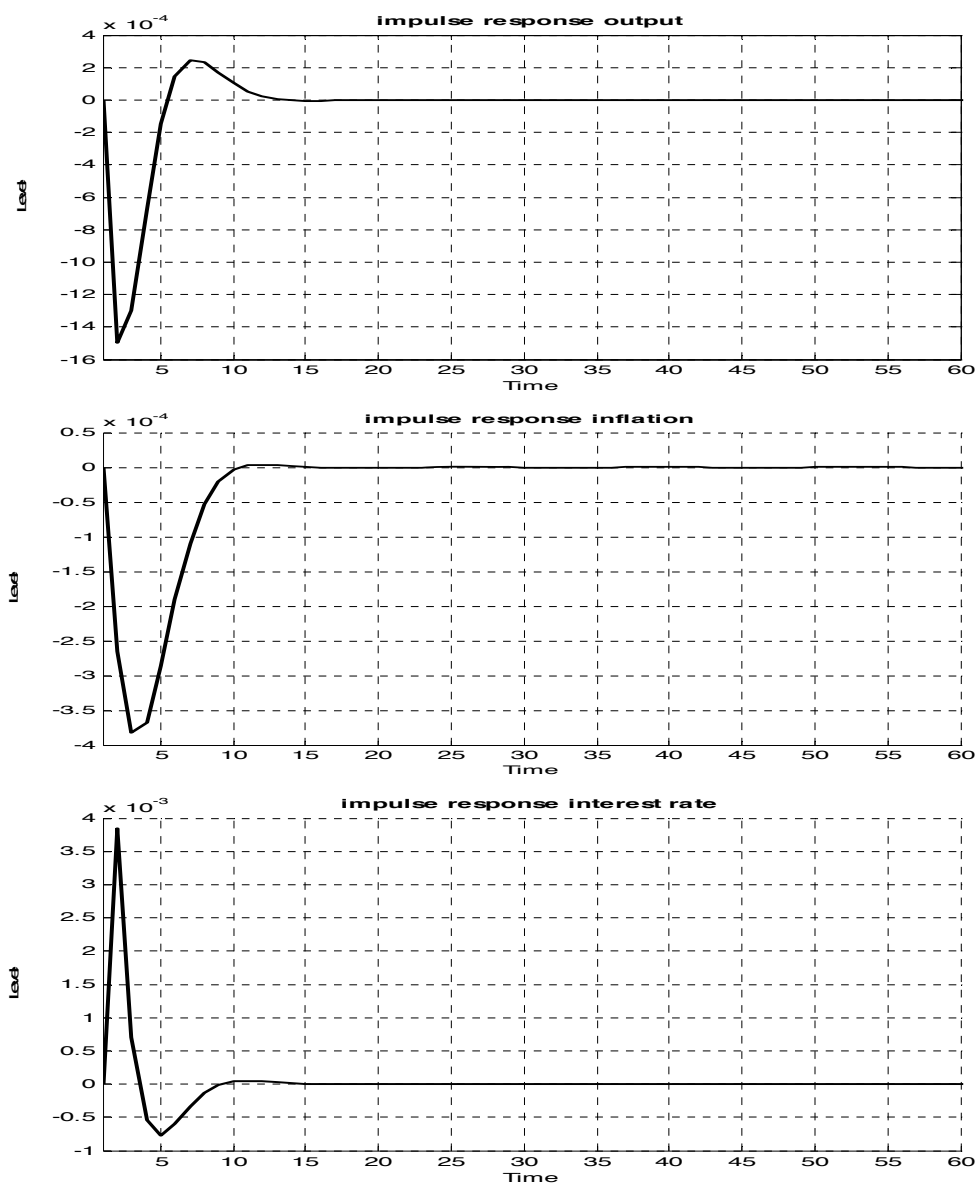
**Figure 3: Mean impulse responses to interest rate shock in the heuristic model**





Note: The dotted lines represent the impulse responses with +/- 2 standard deviations

**Figure 4: Impulse responses to interest rate shock in rational model**



A second contrast between the impulse responses generated by the two models is that the size of the effects of the same interest rate shock in output and inflation is significantly higher in the rational expectations model (compared to the mean effect in the heuristic model). Again this has to do with the low level of inertia in the rational expectations model.

## 2.4 The extended heuristic model

In this section we extend the heuristic model by allowing the inflation forecasters to be heterogeneous. We follow Brazier et al. (2006) in allowing for two inflation forecasting rules. One rule is based on the announced inflation target (as in the previous section); the other rule extrapolates inflation from the past into the future. One may argue that this is quite a different pair of heuristics than in the case of output forecasting. The difference between inflation forecasting and output forecasting is that in the former case there is a central bank that announces a particular inflation target. This target works as an anchor for the forecasts of agents. Such an anchor is absent in the case of output forecasting.

The “inflation targeters” use the central bank’s inflation target to forecast future inflation, i.e.

$$\tilde{E}_t^{tar} = \pi_t^* \quad (17)$$

where as before we set the inflation target  $\pi_t^* = 0$

The “extrapolators” are defined by  $E_t^{ext} = \pi_{t-1}$  (18)

The market forecast is a weighted average of these two forecasts, i.e.

$$\tilde{E}_t \pi_{t+1} = \beta_{tar,t} \tilde{E}_t^{tar} \pi_{t+1} + \beta_{ext,t} \tilde{E}_t^{ext} \pi_{t+1} \quad (19)$$

or

$$E_t \pi_{t+1} = \beta_{tar,t} \pi_t^* + \beta_{ext,t} \pi_{t-1} \quad (20)$$

$$\text{and } \beta_{tar,t} + \beta_{ext,t} = 1 \quad (21)$$

We use the same selection mechanism as in the previous section based on the mean squared forecasting errors produced by the two rules to determine the proportions of agents trusting the inflation target and those who do not trust it and revert to extrapolation of past inflation, i.e.

$$\beta_{tar,t} = \frac{\exp(\gamma U_{tar,t})}{\exp(\gamma U_{tar,t}) + \exp(\gamma U_{ext,t})} \quad (22)$$

$$\beta_{ext,t} = \frac{\exp(\gamma U_{ext,t})}{\exp(\gamma U_{tar,t}) + \exp(\gamma U_{ext,t})} \quad (23)$$

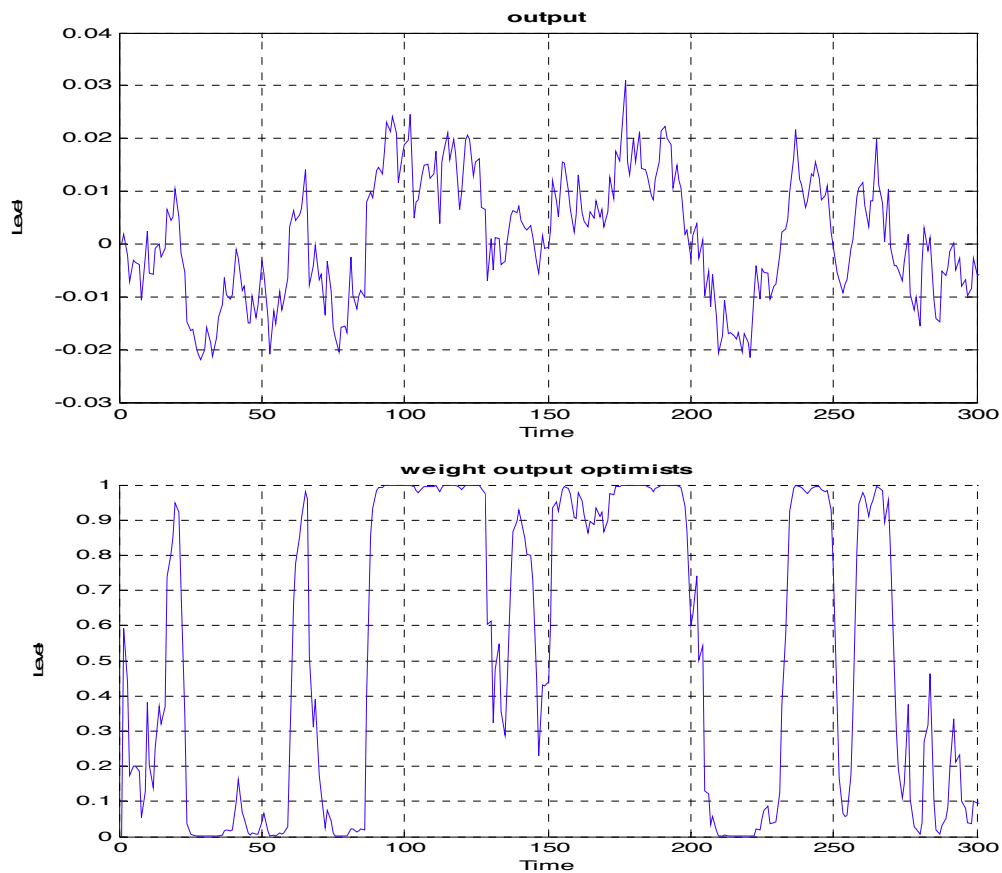
This inflation forecasting heuristics can be interpreted as a procedure of agents to find out how credible the central bank's inflation targeting is. If this is very credible, using the announced inflation target will produce good forecasts and as a result, the proportion of agents relying on the inflation target will be high. If on the other hand the inflation target does not produce good forecasts (compared to a simple extrapolation rule) it will not be used much and therefore the proportion of agents using it will be small.

We calibrated the model using the same parameters as in the previous section. We first show the results in the time domain and then discuss the impulse response functions.

Figure 5 presents the results for the output gap in the time domain. We find the same cycles in the output gap as in the previous section. Again these cycles are related to the waves of optimism and pessimism in the forecasting (second panel in figure 5).

The results concerning the time path of inflation are shown in figure 6. We first concentrate on the second panel of figure 6. This shows the proportion of “extrapolators”, i.e. the agents who do not trust the inflation target of the central bank. We can identify two regimes. There is a regime in which the proportion of extrapolators fluctuates around 50% which also implies that the proportion of forecasters using the inflation target as their guide (the “inflation targeters”) is around 50%. This is sufficient to maintain the rate of inflation within a narrow band of approximately + and – 1% around the central bank's inflation target. There is a second regime though which occurs when the extrapolators are dominant. During this regime the rate of inflation fluctuates significantly more. Thus the inflation targeting of the central bank is fragile. It can be undermined when forecasters decide that relying on past inflation movements produces better forecast performances than relying on the central bank's inflation target. This can occur quite unpredictably as a result of stochastic shocks in supply and/or demand.

**Figure 5: Output gap in the extended heuristic model**

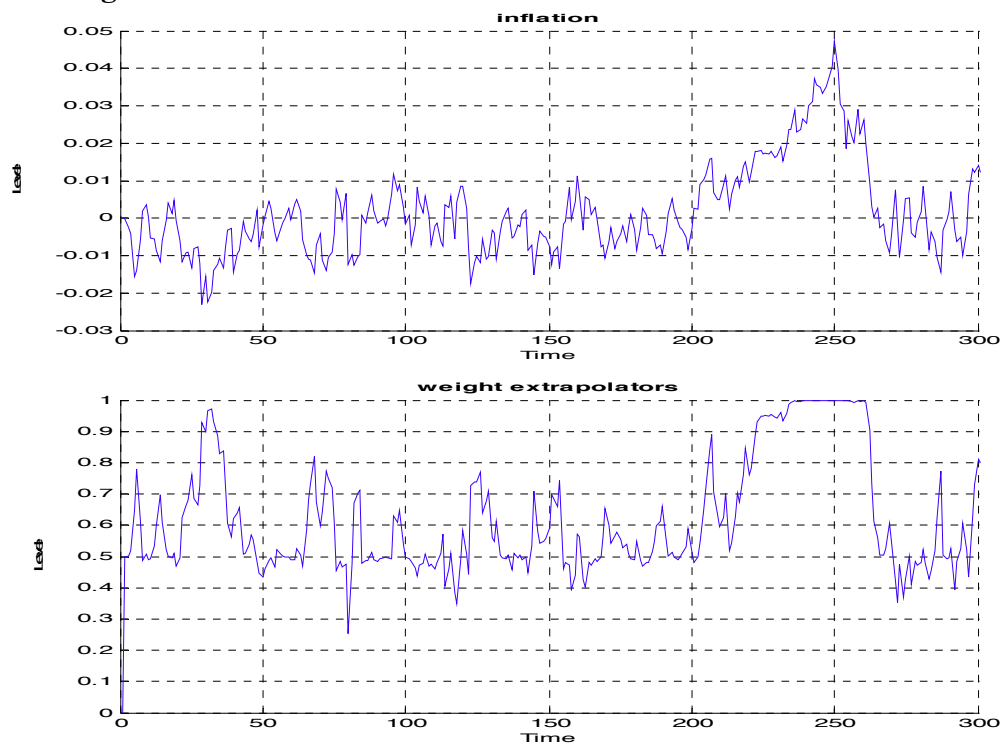


How can the central bank strengthen the inflation targeting regime? The previous simulations assumed an output coefficient of 0.5 in the Taylor equation. This is a value often found in empirical work. It implies that the central bank gives some weight to output stabilization. In a way an output coefficient of 0.5 implies that the central bank deviates from strict inflation targeting. As an alternative the central bank could apply strict inflation targeting, implying that the output coefficient is 0<sup>6</sup>. We show the results of a simulation when the central bank sets the output coefficient equal to zero (strict inflation targeting) in figure 7. We now observe that with strict inflation targeting the first regime dominates, i.e. the rate of inflation stays within the narrow band of +/- 1% most of the time. There are occasional “dérappages” into the second more turbulent regime but these are less frequent and less persistent. This has

<sup>6</sup> This is our interpretation of strict inflation targeting. There is another one which interprets strict inflation targeting to put a zero weight to the output gap in the loss function of the monetary authorities. In this interpretation, the coefficient of the output gap in the Taylor rule could be positive because the central bank may use the information embedded in the output gap to better forecast inflation. See Svensson().

all to do with the fact that a sufficiently large proportion of agents continue to trust the central bank's inflation target as a guide in forecasting.

**Figure 6: Inflation in the extended heuristic model**



**Figure 7: Inflation in the extended heuristic model with strict inflation targeting**

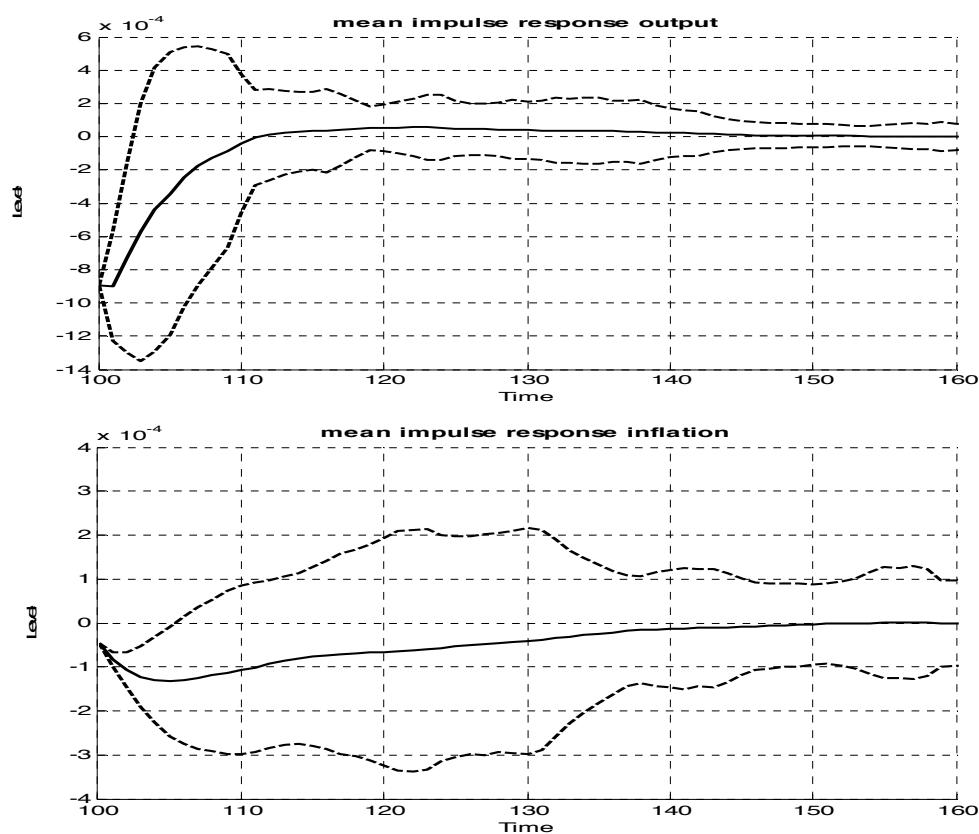


We come back to this issue of the strictness of inflation targeting in section 4 when we subject the choices of the central bank to a more systematic analysis.

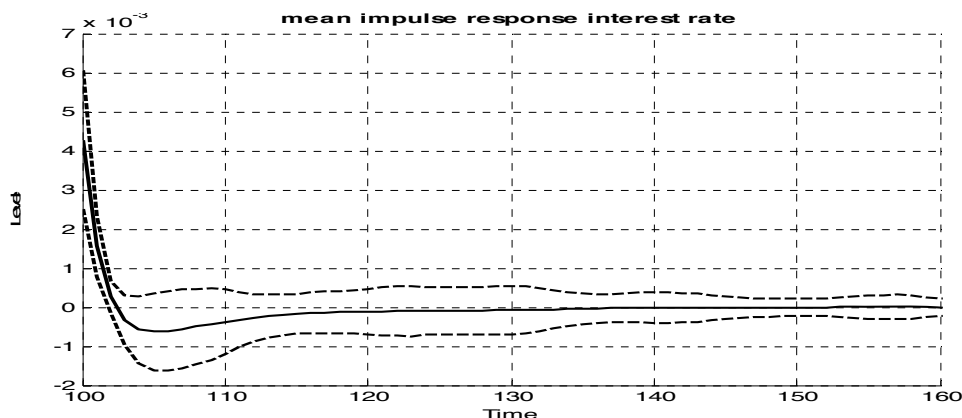
## 2.5 Impulse responses in the extended heuristic model

In this section we present the impulse responses to a positive interest rate shock of one standard deviation. Two results stand out. First the uncertainty surrounding the effects of interest rate shocks is greater and lasts longer than in the simple heuristic model with homogenous inflation forecasting. Second, there is in this extended model considerably more inertia in inflation adjustment than in output adjustment following the interest rate shock. This feature whereby there is more inertia in inflation adjustment than in output adjustment after a shock is routinely found in VAR estimates of interest rate surprises. The inertia generated by the model finds its origin in the evolutionary process inherent in the fitness criterion guiding the selection of forecasting rules<sup>7</sup>.

**Figure 8: Mean impulse responses to an interest rate shock in the extended heuristic model**



<sup>7</sup> A similar result was obtained by Anagastopoulos, et al. (2006)



## 2.6 Market sentiments and impulse responses

An important finding of our model is the dependence of the impulse responses on the initial conditions. This implies that the transmission of shocks depends on the exact timing of these shocks. The reason is that “market sentiment” changes continuously thereby changing the transmission of these shocks.

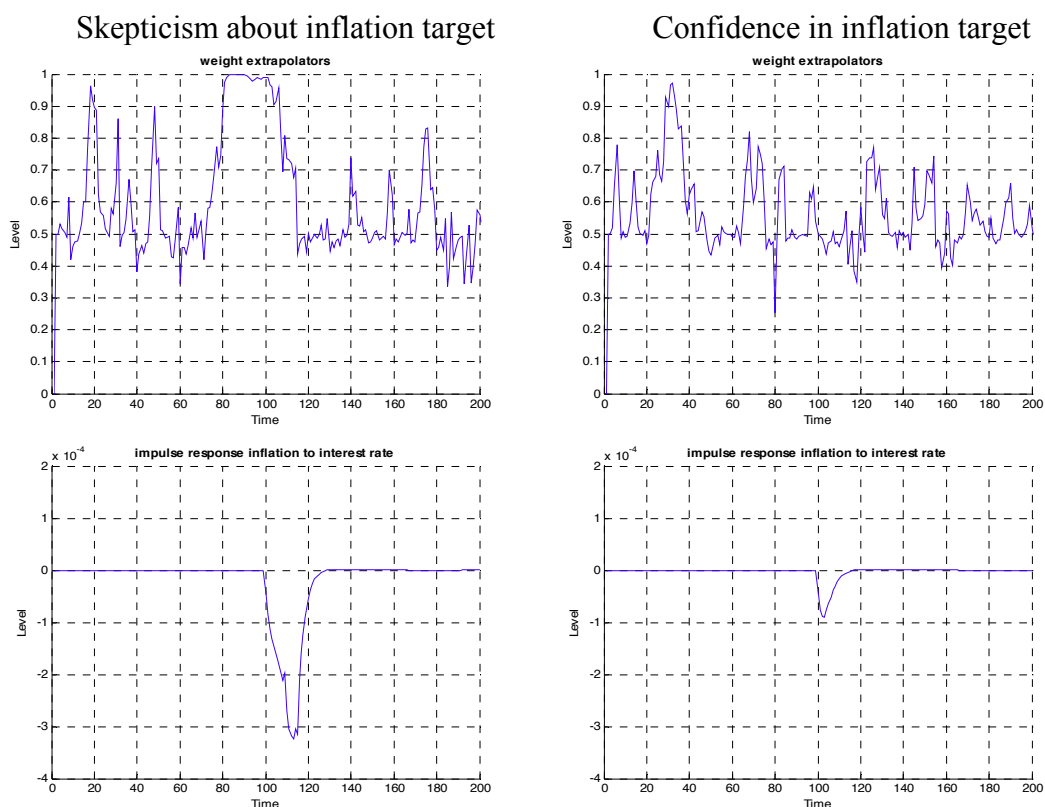
There are two sources of “market sentiment” in the model. One originates with the waves of optimism and pessimism produced by the switching dynamics between optimistic and pessimistic rules in forecasting output. The second one arises from the switches between the two inflation forecasting rules, producing periods of confidence in the inflation target announced by the central bank and periods of skepticism about this inflation target.

In this section we discuss with a few representative examples the nature of this dependence of the transmission mechanism on market sentiments. We start with presenting two impulse responses to the same interest rate shock (a one standard deviation increasing in the interest rate). These two shocks occur when the market sentiments are very different. We show the results in figure 9. The left hand panel shows the impulse response of inflation to an interest rate increase that occurs when the market is skeptical about the announced inflation target. This can be seen by the fact that when the shock occurs (in period 100), almost all agents have become extrapolators, i.e. they have lost confidence in the inflation target. The right hand panel shows the impulse response when the interest rate shock occurs at a time when the weight of extrapolators is low. This is a regime characterized by confidence in the inflation target.

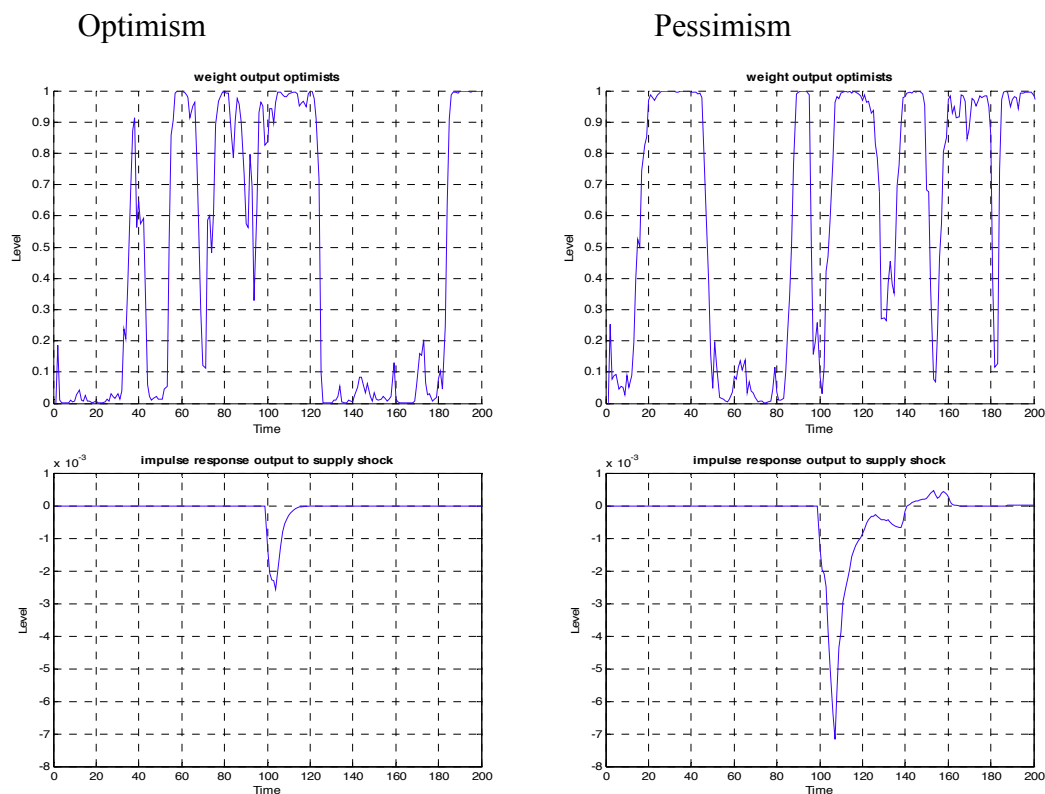
The results are striking. When the market is skeptical about the inflation target the interest rate shock has a substantial effect on inflation, while when the market exhibits confidence in the inflation target the same interest rate increase has only a very small effect on the rate of inflation. Conversely, since the impulse responses are symmetric, a decline in the interest rate has a strong positive effect on inflation when the market is skeptical and a weak effect on inflation when the market is confident in the inflation target. This result is akin to the stabilization bonus obtained in a fully credible inflation targeting regime.

This dependence of the impulse responses on the market sentiments is also obtained when demand and supply shocks occur. We show an example involving a supply shock in figure 10. The left hand panel exhibits the impulse response of output to a supply shock when the market sentiments about output growth are optimistic, while the right hand panel shows the same impulse response when the market sentiments are pessimistic. Again the results are striking. When optimists prevail a negative supply shock has a significantly lower and shorter-lived negative effect on output than when pessimists dominate the market.

**Figure 9: Impulse responses of inflation to interest rate shock (increase)**



**Figure 10: Impulse responses of output to supply shock**



### 3. On the importance of inflation targeting

In the previous section we analyzed the model assuming that the central bank follows an inflation targeting strategy. We assumed two regimes, one in which *all* agents attach full credibility to the inflation target, and another one in which *some* agents believe this target and others do not. In this section we perform another experiment with the model. We will now contrast a regime in which *none* of the agents attach credibility in the central bank's inflation target with one in which some do and others do not attach credibility. This experiment can then be interpreted as mimicking a regime change whereby the central bank moves away from a regime in which the inflation target has no credibility towards a regime of (limited) credibility of inflation targeting.

We model the first regime as one in which agents do not take the inflation target into account when forecasting the future inflation. Thus there is only one inflation forecasting rule that all agents use, i.e. the extrapolative rule as represented by

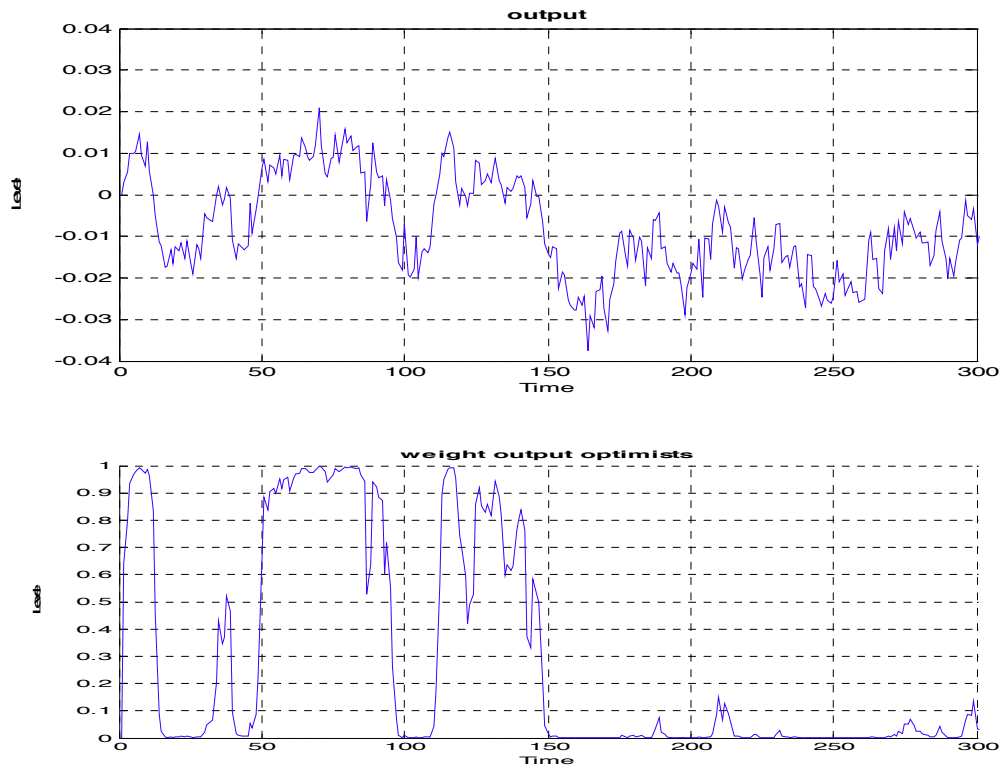
equation (18). The second regime corresponds to what we have called the extended heuristic model in which we use the two inflation forecasting rules (17) and (18).

We first present the results of simulating these two regimes in the time domain using the same calibration as in the previous sections. Figures 11 and 12 show the evolution of the output gaps and the fraction of optimists in the two inflation targeting regimes. The results are striking. In the regime of complete absence of a credible inflation target the cycles in the output gap are much longer than in the regime of imperfectly credible inflation targeting. These longer cycles in the former regime are related to the fact that the waves of optimism and pessimism are longer and more protracted. This difference is also evident from the autocorrelation coefficients of the output gap. In the first regime without inflation targeting this autocorrelation coefficient is 0.44 while it is only 0.29 in the regime with imperfectly credible inflation targeting. In addition, we find that the standard deviation of the output gap is 1.6% in the first regime versus 1.1% in the second regime. Thus the introduction of (imperfectly) credible inflation targeting reduces the volatility of the output gap and makes the waves in the output gap shorter (it reduces the inertia as measured by the autocorrelation coefficient).

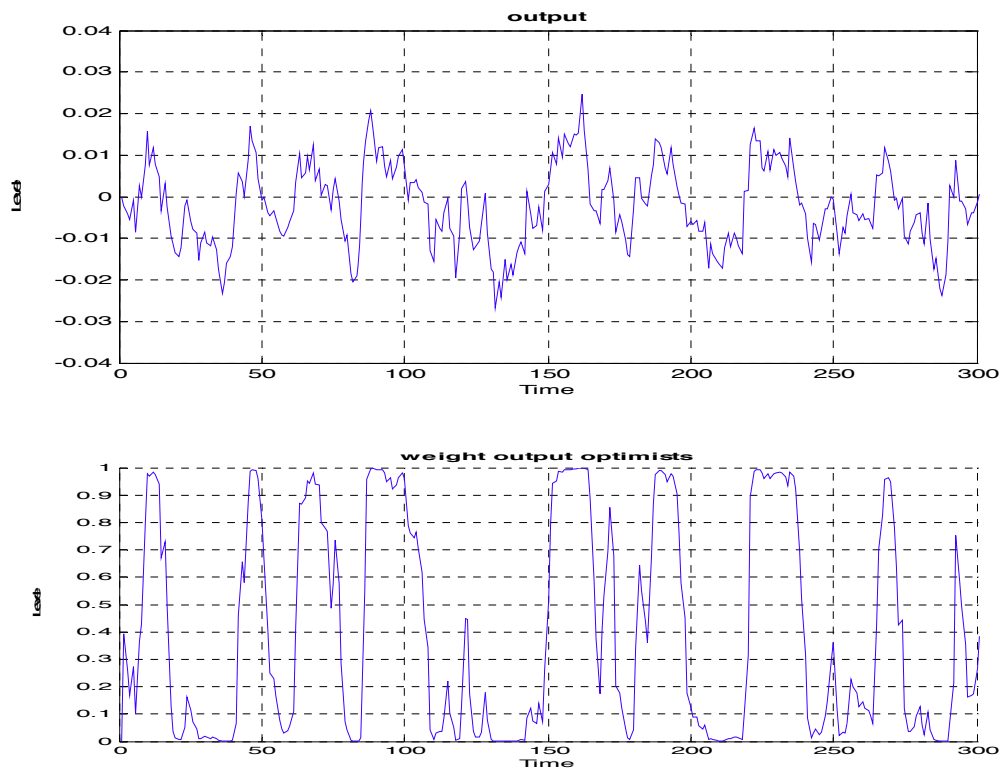
The reason why we obtain such a pronounced difference in output stabilization is related to the fact that in the absence of a credible inflation target, the rate of inflation is subjected to stronger movements than in the regime of imperfectly credible inflation targeting. We show the contrast in figures 13 and 14. It can clearly be seen that inflation in the former regime fluctuates much more than in the latter regime. (The standard deviation of inflation in the former regime is 1.98% versus 0.96% in the latter regime).

The higher volatility of inflation in the regime where the inflation target lacks all credibility forces the central bank to adjust the interest rate in a more aggressive way than in the regime with imperfectly credible inflation targeting. As a result, the absence of a credible inflation target produces more volatility in the interest rate. This increased volatility becomes a source of additional volatility in the output gap. We conclude that the establishment of an inflation targeting regime (even if imperfectly credible) stabilizes not only the rate of inflation but also the business cycle. This remarkable property is obtained even in a world where agents cannot form expectations rationally.

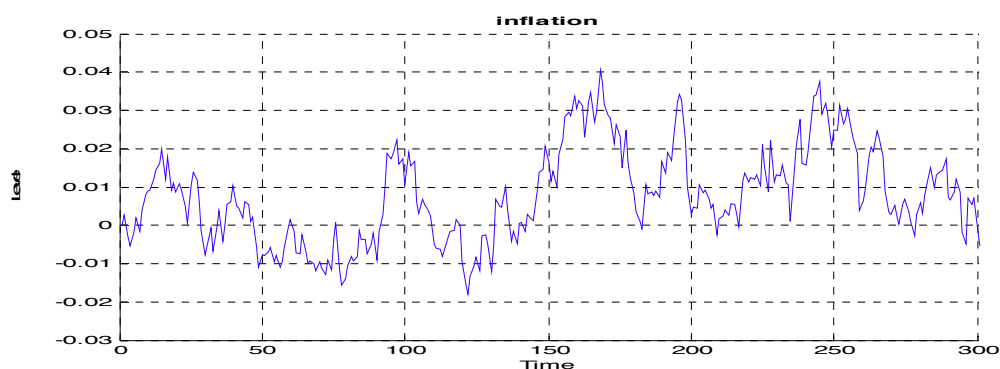
**Figure 11: Output in a regime without credible inflation target**



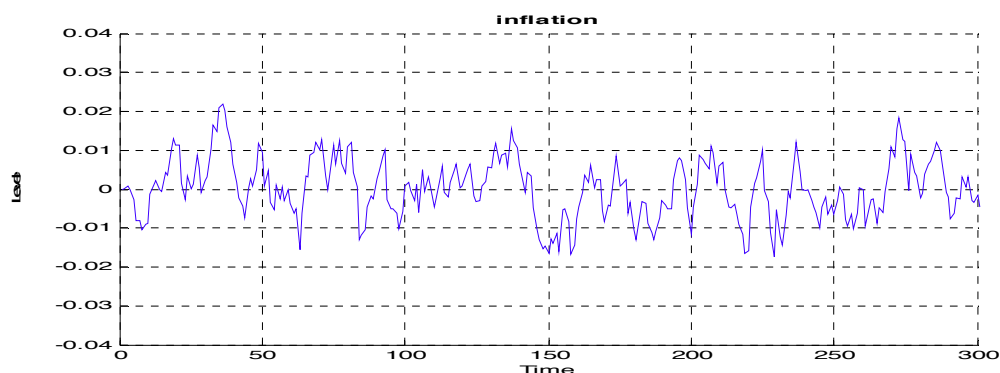
**Figure 12: Output in a regime with (imperfectly) credible inflation target**



**Figure 13: Inflation in a regime without credible inflation target**



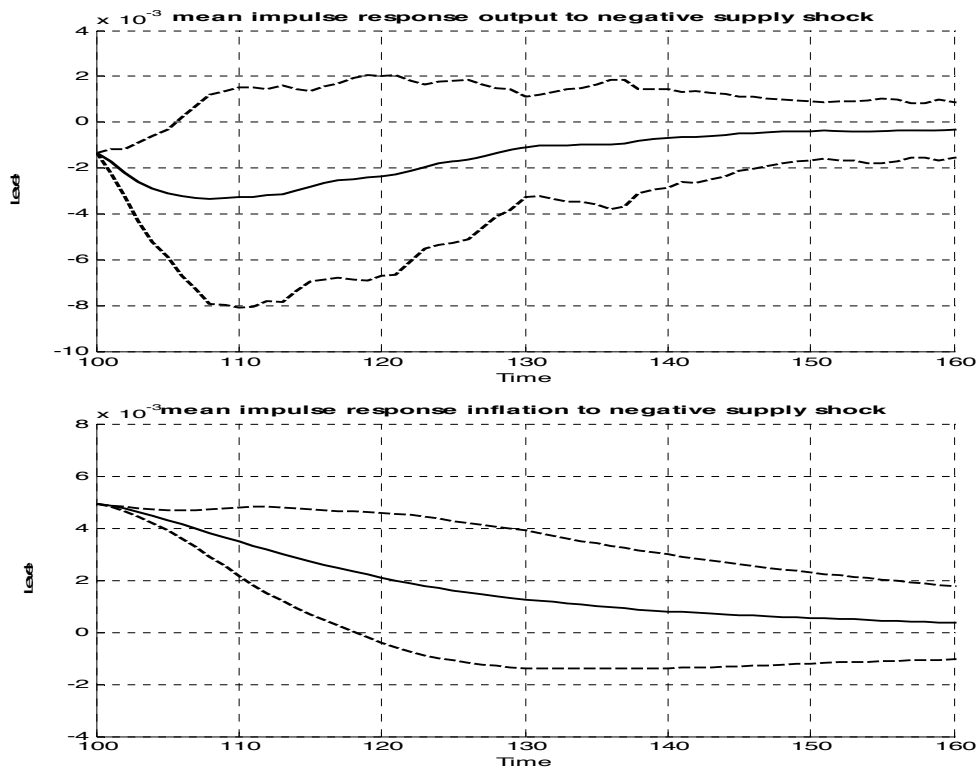
**Figure 14: Inflation in a regime with (imperfectly) credible inflation target**



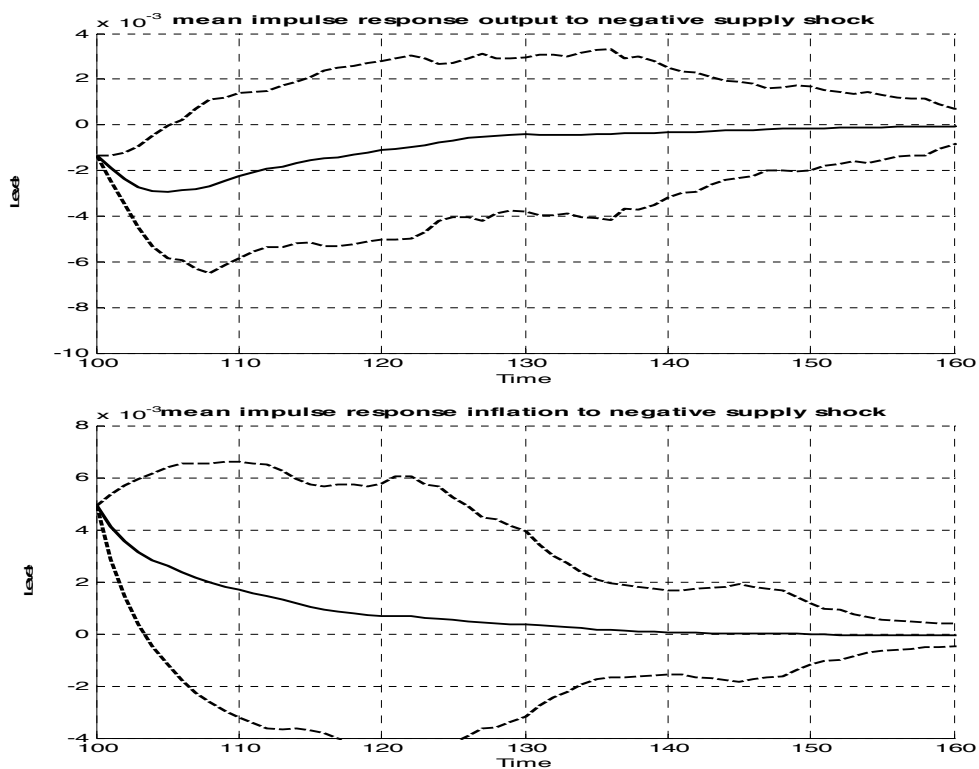
Next we compare the impulse responses of output and inflation in the two regimes. We focus on the responses to a (negative) supply shock. These impulse responses are shown in figures 15 and 16. We find that in the absence of a credible inflation target, the negative supply shock leads to a significantly longer adjustment in output and inflation than in the regime of imperfectly credible inflation targeting. Put differently, a negative supply shock leads to a less protracted effect on output and inflation in a regime of imperfectly credible inflation targeting than in the absence of a credible inflation target. In this sense inflation targeting helps in reducing the impact of a negative supply shock<sup>8</sup>.

<sup>8</sup> This has also been found in DSGE-models and is sometimes called the stabilization bias. See Gaspar, et al. (2006)

**Figure 15: impulse responses of output and inflation (absence of credible inflation target)**



**Figure 16: impulse responses of output and inflation (with imperfectly credible inflation target)**



#### 4. Trade-offs between inflation and output variability

We have seen that the central bank can reduce inflation variability by applying a stricter inflation targeting regime. This comes at a price in terms of output volatility though. We analyze this tradeoff in the context of the extended heuristic model and compare it to the tradeoff in a rational expectations model.

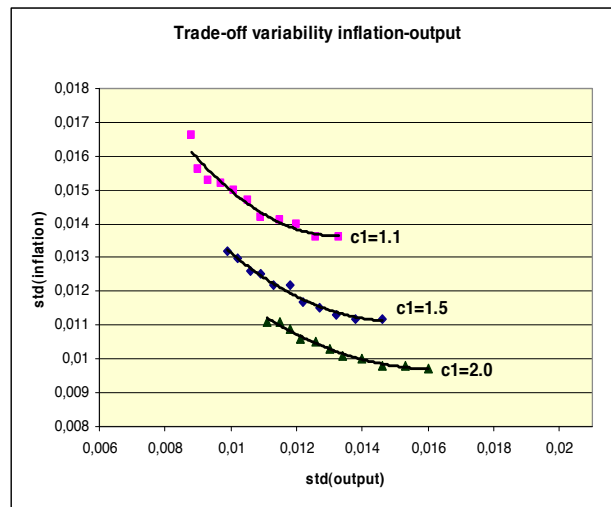
Figure 17 presents the tradeoff in the heuristic model and figure 18 in the rational expectations model. These tradeoffs are obtained by increasing the output coefficient ( $c_2$ ) in the Taylor rule from 0 to 1 and then computing the standard deviations of inflation and output gaps for these different values of  $c_2$ . These standard deviations are then plotted in figures 17 and 18. We have computed three tradeoffs corresponding to three different values of the inflation coefficient in the Taylor rule, i.e.  $c_1 = 1.1$ ,  $c_1 = 1.5$ ,  $c_1 = 2.0$ . These tradeoffs show that when the central bank that wishes to reduce inflation volatility by applying stricter inflation targeting (an increasing value of  $c_2$ ) will have to allow for more output variability. We note from figures 17 and 18 that the trade-off improves when  $c_1$  increases, i.e. when the central bank reacts more forcefully to an inflation upsurge, it can achieve both lower inflation and output variability<sup>9</sup>.

Comparing figures 17 and 18 we observe a few differences. First, the tradeoffs appear to be steeper in the heuristic model as compared to the rational expectations model. This comes from the fact that the application of more flexibility in inflation targeting (a high  $c_2$ ) reduces output variability more in the rational model than in the heuristic model. Thus reducing output variability in the rational model comes at a lower cost in terms of inflation variability. We come back to this feature when we apply a welfare analysis. Second, the value of  $c_1$  has a significantly lower effect on improving the tradeoff in the rational expectations model as compared to the heuristic model. Put differently in a rational expectations world a more forceful reaction of the central bank to an inflation surge (a higher  $c_1$ ) does not improve the trade-off significantly.

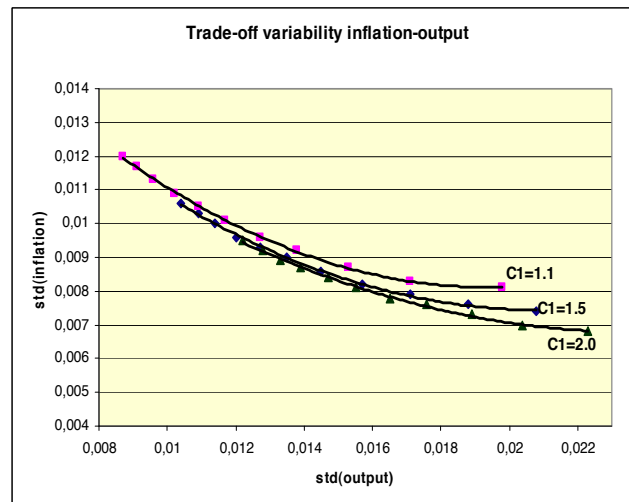
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<sup>9</sup> A similar result on the importance of strict inflation is also found in Gaspar, Smets and Vestin(2006) who use a macromodel with statistical learning.

**Figure 17: Trade-offs in the extended heuristic model**



**Figure 18: Trade-offs in the rational model**



## 5. Welfare analysis

In this section we compare welfare obtained in both models. We assume a loss function of the central bank applying equal weights to inflation and output variability. This approach is not fully satisfactory. Ideally we should specify the central bank's utility function from the start and derive the optimal policy. We leave that for further research.

We obtain the welfare losses for different values of  $c_1$  and  $c_2$  as shown in figure 19. A comparison of the welfare losses in the two models leads to the following interesting observation. In a rational expectations model a greater flexibility in inflation targeting (increasing  $c_2$ ) leads to significantly larger welfare gains than in the heuristic model.

The reason is that the rational expectations model implicitly assumes commitment. As a result, increasing the degree of flexibility of inflation targeting leads to a large decline in output variability that is not fully compensated by an increase in inflation variability.

In a way it can be said that this difference has to do with the fact that implicitly we use a different degree of commitment in the two models. In the rational model we have full commitment, while in the heuristic model we allow agents to test the commitment of the central bank by the switching rule between targeters and extrapolators.

In order to make the two models comparable with respect to the degree of commitment, we used the simple heuristic model in which all inflation forecasts are based on the central bank's inflation target and compared the trade-offs and welfare numbers obtained in that model with those obtained in the rational model. We show the results in figures 20 and 21

**Figure 19: Welfare losses in the rational and the extended heuristic models**

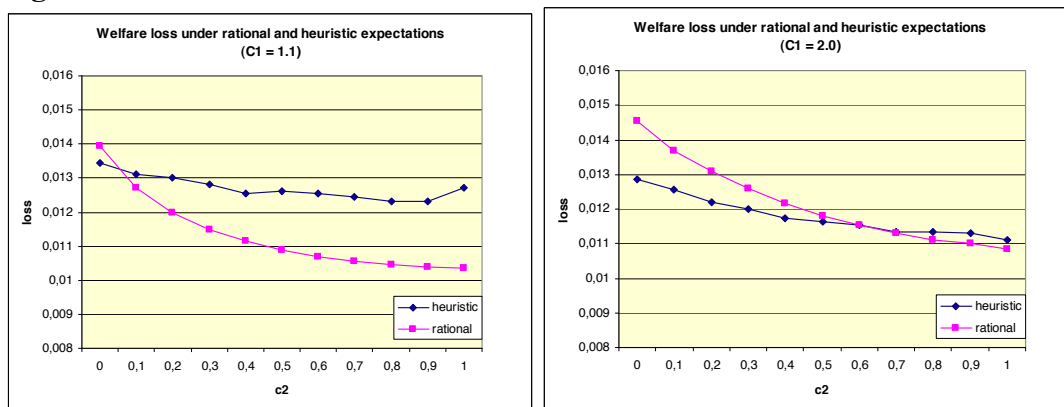
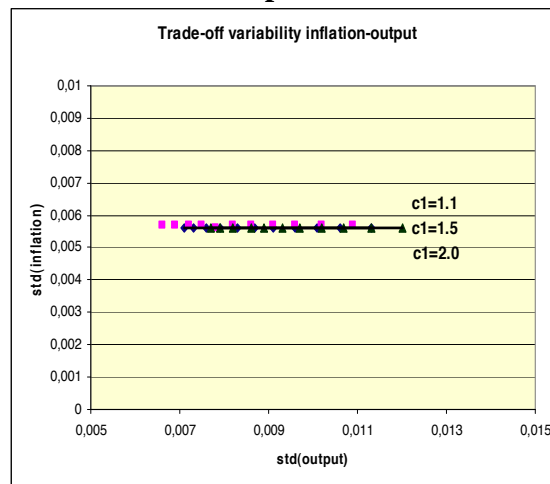


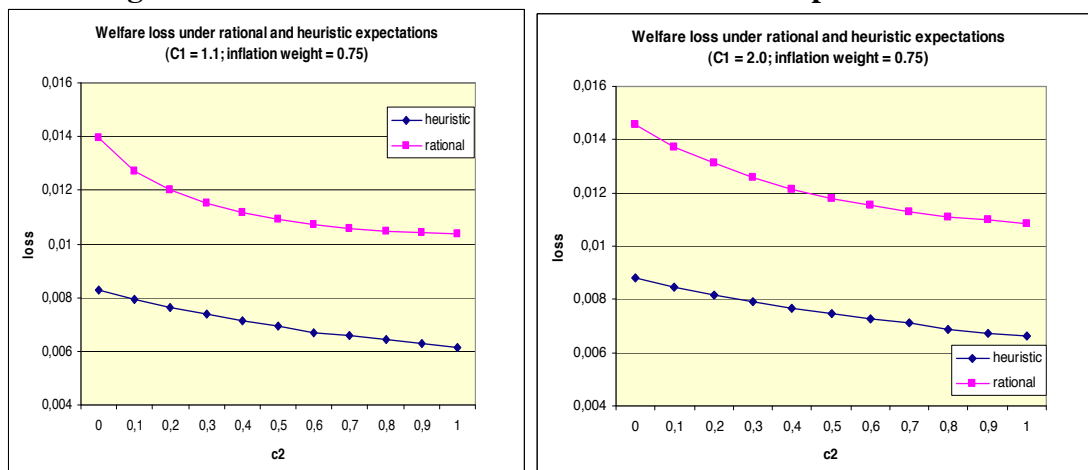
Figure 20 presents the trade-off between inflation and output variability in the simple heuristic model. We observe that the trade-off is close to a horizontal line, i.e. the central bank can increase the degree of flexibility of its inflation targeting without significant loss in terms of higher inflation variability. The reason is that all agents are confident in the usefulness of the inflation target as an anchor for their forecasting and nobody wants to switch to another rule. As a result, the central bank can exploit its credibility by increasing the intensity of output stabilization without being punished with higher inflation variability.

Figure 21 compares the welfare losses in the two models. Two results are worth noting. First, the welfare losses are uniformly lower in the heuristic model than in the rational model. The reason has to do with the fact, noted earlier, that the rational model used here produces a greater amount of variability than the heuristic model. Second, we now find that increasing the degree of flexibility of inflation targeting reduces the welfare losses in a similar degree in both models.

**Figure 20: Trade-offs in the simple heuristic model**



**Figure 21: Welfare losses in the rational and the simple heuristic model**



## 6. Endogenous and exogenous inertia

As pointed out earlier, DSGE-models introduce inertia into the model by imposing lags into the transmission mechanism, the logic of which comes mostly from outside the model. To give an example, Calvo pricing in which firms are constrained to adjust

prices instantaneously (Christiano, Eichenbaum and Evans (2001)) is routinely imposed in these models. It is clear, however, that such a restriction comes from outside the logic of the model. In a world where everybody understands the model and each other's rationality, agents would want to go immediately to the optimal plan using the optimal price. They would not want to accept such a restriction<sup>10</sup>.

We could call the inertia introduced in the DSGE-model an exogenously created inertia. In contrast, the heuristic model presented here is capable of generating inertia without imposing lags in the transmission process. This could be called endogenous inertia. We illustrate this difference by analyzing the heuristic and the rational model in the absence of lags in the transmission process in the demand and the supply equations. We achieve this by setting  $a_1 = 1$  in equation (1) and  $b_1 = 1$  in equation (2). We then applied the same i.i.d. shocks in both the heuristic and the rational model and computed the autocorrelation coefficients of the simulated series of output gaps and inflation. We show the results in table 1. We observe that the heuristic model produces inertia (positive autocorrelation) in the output gap and in inflation even if there are no lags in the transmission of shocks. Our rational model produces no inertia in the output gap and in inflation.

Table 1 also shows the autocorrelation coefficients obtained in models that assume lags in the transmission. These coefficients are obtained when we set  $a_1 = 0.5$  in equation (1) and  $b_1 = 0.5$  in equation (2). These are also the numerical values assumed in all the simulations reported in the previous sections. We now observe that inertia in the output gap and in inflation increases in both models. However, it can be concluded that all of the inertia obtained in the rational model is the result of the lags in the transmission process. This is not the case in the heuristic model where most of the inertia is produced endogenously.

We also note from table 2 that even when the coefficients  $a_1$  and  $b_1$  of the forward looking variables of the model are set at 0.5, the rational model produces less inertia than the heuristic model. We explore the sensitivity of the autocorrelation coefficients

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<sup>10</sup> The use of Calvo-pricing rules is often justified by invoking institutional restrictions that limit the freedom of action of individual firms. The question arises why rational and perfectly informed agents would accept institutions that limit their freedom to set optimal plans. After all, it is against their own interest to accept such limitations. It is not only against the interests of the firms, but also of consumers and workers, who in the rational macroeconomic models are agents who perfectly understand the world and their own interests and will always want to optimize their utilities. Any limitation on their optimizing behaviour reduces their welfare.

to these parameters more exhaustively in figure 22. This shows the autocorrelation coefficients as a function of  $a_1$  and  $b_1$ . We observe that in the heuristic model the autocorrelation coefficients are not very sensitive to the  $a_1$  and  $b_1$ . This contrasts a great deal with the results of the rational model, where the sensitivity is very high. When  $a_1$  and  $b_1$  are close to 1 (i.e. no or weak lags in the transmission) the autocorrelation coefficients are very low (very low inertia). In order to produce inertia in the rational model which is of the same magnitude as in the heuristic model,  $a_1$  and  $b_1$  must exceed 0.5.

This difference between the two models is quite fundamental. In the rational model there is (due to its linearity) no uncertainty about how the shock is transmitted in the model. Thus in the absence of lags in the transmission, agents will immediately find the optimal levels of output and inflation with minimal inertia<sup>11</sup>. In order to produce the required inertia, lags in the transmission preventing instantaneous adjustment to the optimal plan, are necessary. In the heuristic model, agents do not fully understand how the shock will be transmitted. As a result they follow a procedure (heuristics together with a selection mechanism) that functions very much like a “trial and error” mechanism aimed at revealing the information about shocks and the transmission process. This is a slow process that also uses backward evaluation processes. It generates an endogenous inertia into the model.

Critics of the heuristic model presented here may argue that the comparison between the rational and the heuristic model is unfair for the rational model. Indeed the heuristic model generates inertia because the evaluation and selection process of the different heuristics is backward looking. This is the reason why the heuristic model does not need lags in the transmission process to generate inertia. However, we claim that this evaluation and selection process can only be backward looking, and as a result, the lags that are present in the heuristic model are completely within the logic of that model. This contrasts with the lags introduced in the rational model: they come from outside the model.

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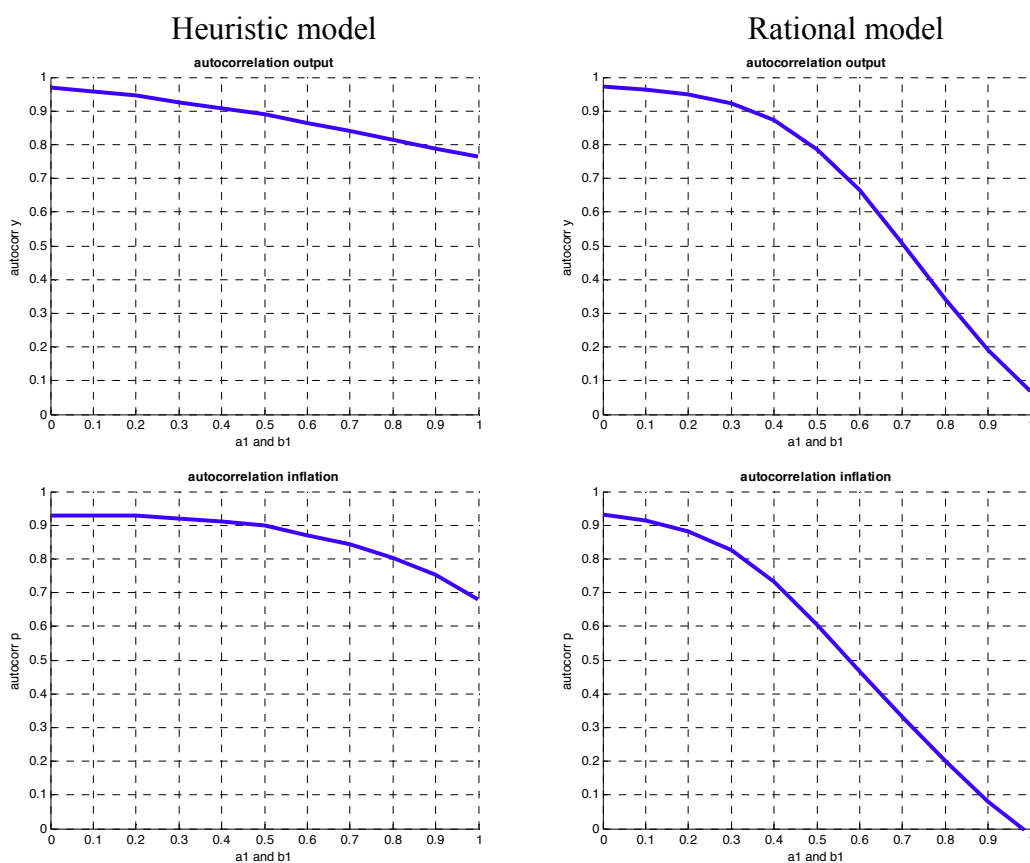
<sup>11</sup> There could still be inertia in output which is produced because agents smooth consumption over time after a productivity shock. This effect is very weak though in the model used here.

Table 2 : Autocorrelation coefficients in output gap and inflation

No lags in transmission		
	heuristic model	rational model
output gap	0.77	0.07
inflation	0.69	-0.02
Lags in transmission		
	heuristic model	rational model
output gap	0.89	0.79
inflation	0.90	0.61

Note: the autocorrelation coefficients are the averages obtained from simulating the model 1000 times, each time over 1000 periods.

Figure 22: Autocorrelation coefficients of output gap and inflation



Note: see note of table 1; we have always set  $a_1 = b_1$

## 7. Sensitivity analysis

In this section we analyse how sensitive the results are to different numerical values of the “learning parameters” in the model. These are the parameters describing how agents use and select forecasting rules. There are three such parameters in our model. First, there is the divergence between the optimists and pessimist beliefs. We will call this the divergence parameter, which we define as  $2g$  (remember that  $g$  is the bias of the optimists and  $-g$  is the bias of the pessimists).

Second, there is the memory agents have when calculating the performance of their forecasting. This was represented by the parameter  $\omega_k$  in equations (9)-(10) and is a series of declining weights attached to past forecast errors. We define  $\omega_k = (1 - \rho)\rho^k$  (and  $0 \leq \rho \leq 1$ ). The parameter  $\rho$  can be interpreted as a measure of the memory of agents. When  $\rho = 0$  there is no memory; i.e. only last period’s performance matters in evaluating a forecasting rule; when  $\rho = 1$  there is infinite memory.

Finally, there is the parameter  $\gamma$  which measures the intensity with which agents are willing to switch to a better performing rule (see equations (11)-(12)).

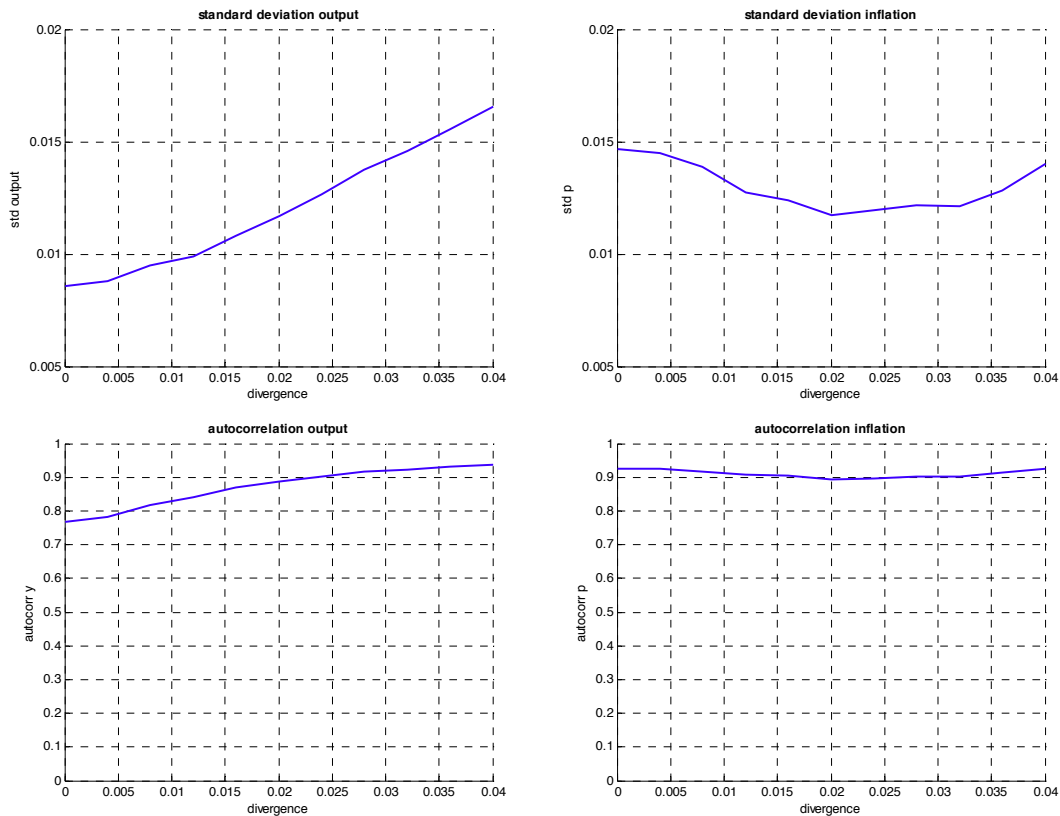
We discuss the sensitivity of the results with respect to these parameters by showing how these parameters affect the volatility of inflation and output, and the degree of inertia (autocorrelation) in these variables.

### 7.1 Sensitivity to divergence in beliefs

The upper panels of figure 23 show how the volatility of output and inflation depends on the degree of divergence in beliefs in forecasting output. We observe that when divergence increases the volatility of output increases substantially. No such increase occurs with inflation which is not surprising as the divergence parameter relates to differences in beliefs about future output.

The lower panels of figure 23 indicate that increasing divergence tends to increase inertia in output (autocorrelation), with little effect on inflation inertia.

**Figure 23: Standard deviation and autocorrelation of output gap and inflation**



Note: the standard deviations and autocorrelation coefficients are the averages obtained from simulating the model 1000 times, each time over 1000 periods.

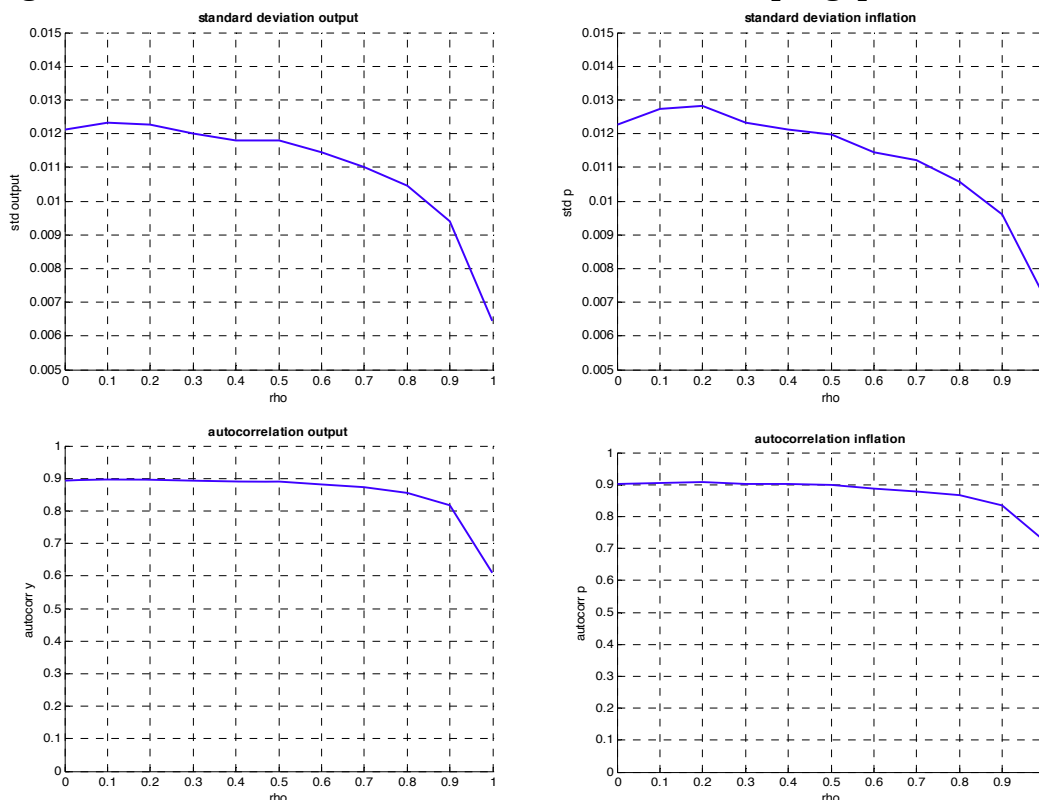
### 7.2 Sensitivity to memory

The memory agents use when they evaluate their past performance, plays an important role in the dynamics of the model. This is illustrated by figure 24. The upper part shows the volatility of output and inflation for different values of the memory parameter ( $\rho$ ). It is striking to find that with increasing memory the volatility of these variables declines significantly. Note however that the relationship is non-linear. One needs a large value of  $\rho$  for the volatility to start declining. In the simulations presented in the previous sections we set  $\rho=0.5$ . The volatility obtained for this parameter value is very close to the volatility obtained when  $\rho=0$  (i.e. when agents have no memory and only the performance of the last period).

We obtain similar results with the autocorrelation coefficients of output and inflation. For low and medium values of  $\rho$  the autocorrelation coefficients are relatively constant. One needs a sufficiently large value of the memory parameter to reduce the

autocorrelation coefficients significantly. We conclude that long memory tends to stabilize output and inflation and to reduce inertia in these variables.

**Figure 24: Standard deviation and autocorrelation of output gap and inflation**



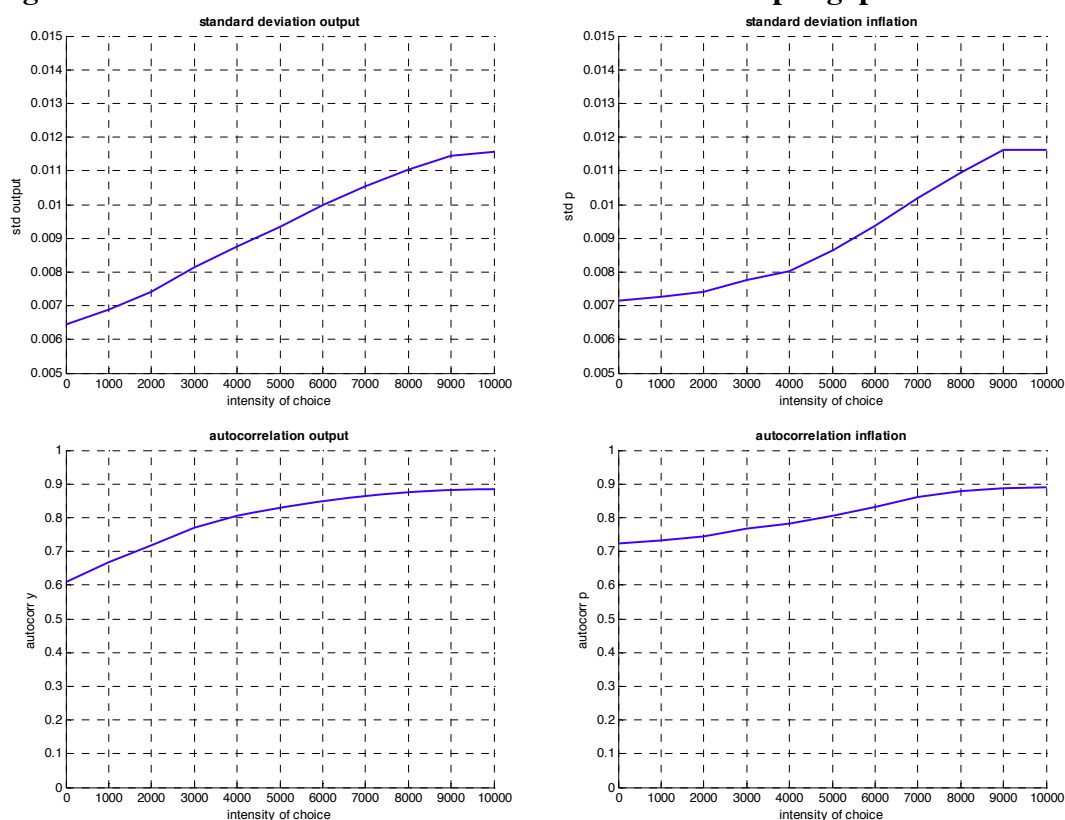
Note: the standard deviations and autocorrelation coefficients are the averages obtained from simulating the model 1000 times, each time over 1000 periods.

### 7.3 Sensitivity to intensity of choice

The intensity of choice parameter controls the degree with which agents switch from one rule to the other when the performance of the forecasting rules change. In general we find that, as this parameter increases, volatility and inertia tend to increase. This is illustrated in figure 25. The upper panel shows the volatility of output and inflation as a function of the intensity of choice parameter. We observe a clear positive relation. The lower panel shows how the autocorrelation coefficients increase when intensity of choice is increased.

We conclude that as agents react more forcefully to changes in performance of their forecasting rules, the volatility of output and inflation and their inertia increases.

**Figure 24: Standard deviation and autocorrelation of output gap and inflation**



Note: the standard deviations and autocorrelation coefficients are the averages obtained from simulating the model 1000 times, each time over 1000 periods.

## 8. Conclusion

DSGE-models provide a coherent framework of macroeconomic analysis. This coherence is brought about by restricting acceptable behaviour of agents to dynamic utility maximization and rational expectations. These features explain the intellectual appeal of these models and their recent success in academic circles and among policymakers.

The problem of the DSGE-models (and more generally of macroeconomic models based on rational expectations) is that they assume extraordinary cognitive capabilities of individual agents. Recent developments in other disciplines including psychology and brain science document that individual agents struggle with limited cognitive abilities, restricting their capacity to understand the world. As a result, individual agents use small bits of information and simple rules to guide their behaviour.

We have used these new insights to extend the DSGE-model framework to an environment in which agents use simple rules to forecast output and inflation. In order to provide discipline in the use of these rules we have introduced a mechanism that allows for the selection of those rules that are more profitable than others.

The ensuing “heuristic model” produces a number of results that distinguishes it from the rational expectations models. First, the heuristic model is capable of generating endogenous cycles based on waves of optimism and pessimism. This dynamics is akin to what Keynes called animal spirits. Second, in contrast to the rational expectations DSGE-models the inertia in output and prices is generated internally in the model, instead of being “imported”. Third, due to its non-linearity, the heuristic model produces a degree of uncertainty about the transmission of monetary policy shocks that is different from the uncertainty obtained in DSGE-models. In the latter linear models, uncertainty about the effects of monetary policy shocks arises only because of the lack of precision in the estimation of the structural parameters of the model. In the heuristic model there is an additional dimension to uncertainty. This is that the same policy shock can have different effects depending on the state of the economy, including the degree of optimism and pessimism agents have about the future. True, DSGE-models can potentially produce similar results. However, these have routinely been excluded by linearizing an otherwise non-linear model.

The success of DSGE-model has much to do with the story it tells about how the macroeconomy functions. This is a story in which rationality of superbly informed and identical agents reigns. Shocks from the outside occur continuously forcing these agents to re-optimize all the time. Unfortunately and inexplicably, the outside world imposes restrictions on this behaviour creating distortions and departures from optimality. It also generates cycles in output and inflation. This in turn creates a stabilizing responsibility for the central bank.

We have questioned this story by presenting an alternative one. This is a story in which agents do not understand the model well, and use a trial and error strategy to discover its underlying logic. Such a model generates cycles endogenously. Thus in contrast with the DSGE-world where the shocks come from outside, in the heuristic world some shocks are generated within the model.

There is another dimension in the difference between the two models. In his famous AER-article Hayek(1945) stressed that individuals have only very small parts of the available information in their brains. No individual can ever hope to understand and to process the full complexity of the world in which he lives. That's why markets are so important. They are vehicles that efficiently aggregate information that is spread around in society. Our model is in the logic of this Hayekian view. This is the logic that produces cycles when markets aggregate the different and incomplete pieces of information individuals use when forecasting the future.

The research presented in this paper should be considered to be preliminary. In order to be convincing as an alternative modeling strategy, a rigorous empirical evaluation of the model will be necessary, whereby the predictions of the model are confronted with the data. In addition, the menu of heuristics which is extremely small in this paper will have to be broadened so that the selection of the "fittest" rules can occur using a wider pool of possible rules.

## Appendix A: parameter values of the calibrated model

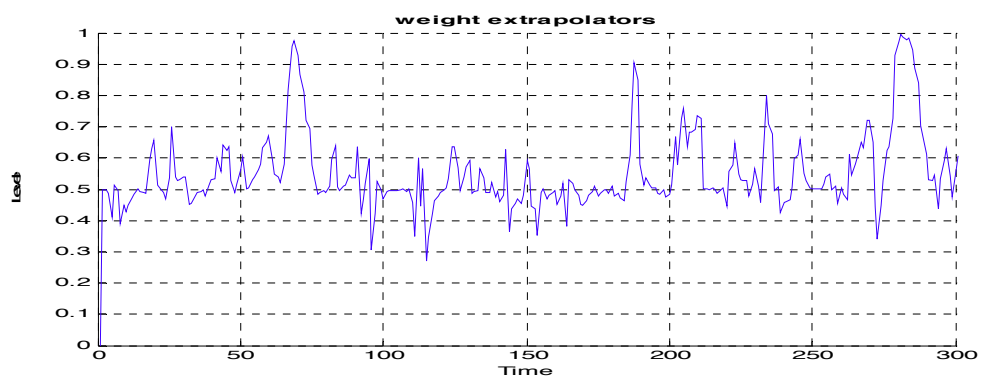
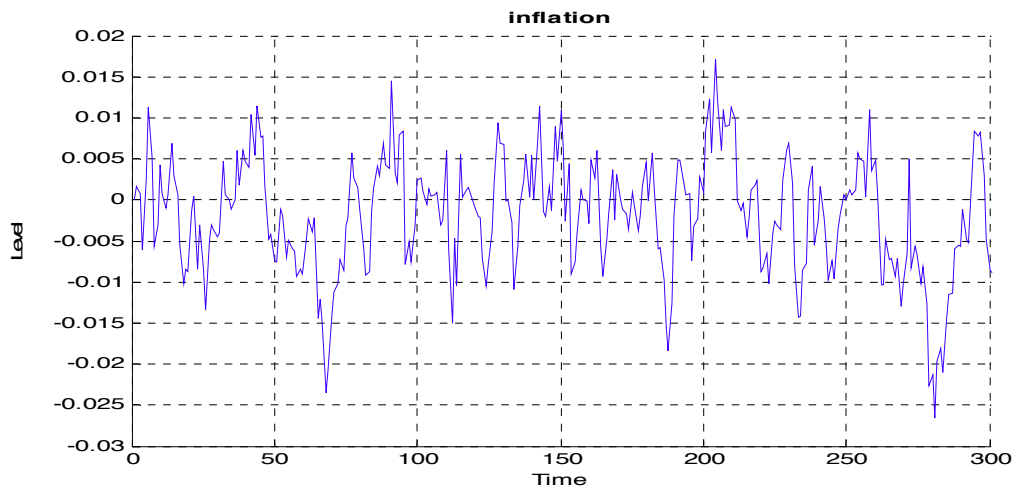
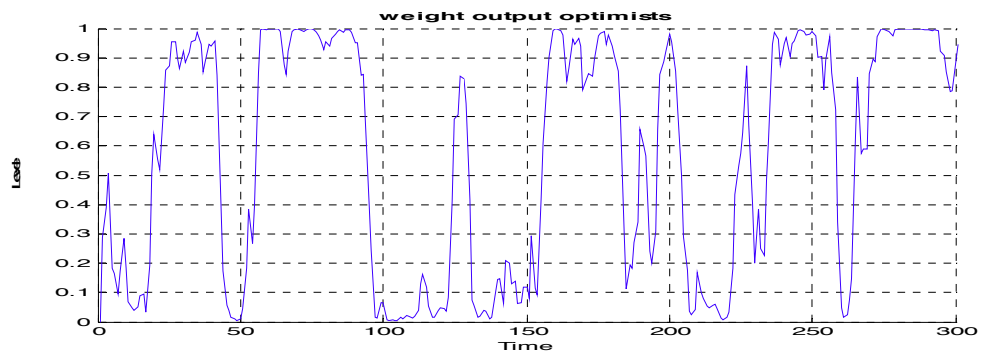
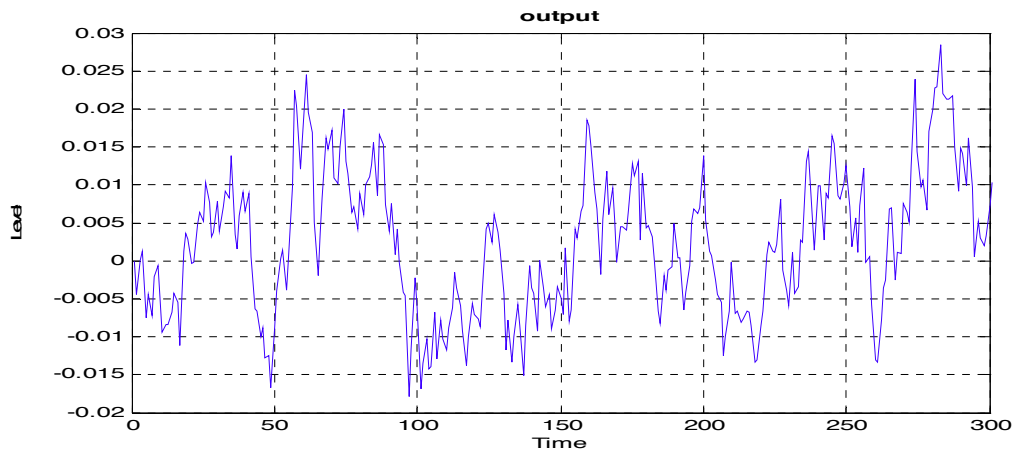
### Heuristic model

```
pstar = 0;           % the central bank's inflation target
a1 = 0.5;           %coefficient of expected output in output equation
a2 = -0.2;         %a is the interest elasticity of output demand
b1 = 0.5;           %b1 is coefficient of expected inflation in inflation equation
b2 = 0.05;         %b2 is coefficient of output in inflation equation
c1 = 1.5;           %c1 is coefficient of inflation in Taylor equation
c2 = 0.5;           %c2 is coefficient of output in Taylor equation
c3 = 0.5;           %interest smoothing parameter in Taylor equation
g = 0.01;          %output forecasts optimists
gamma = 10000;     %switching parameter gamma in Brock Hommes
sigma1 = 0.005;    %standard deviation shocks output
sigma2 = 0.005;    %standard deviation shocks inflation
sigma3 = 0.005;    %standard deviation shocks Taylor
rho=0.5;           %rho measures the speed of declining weights omega in mean
                    squares errors
```

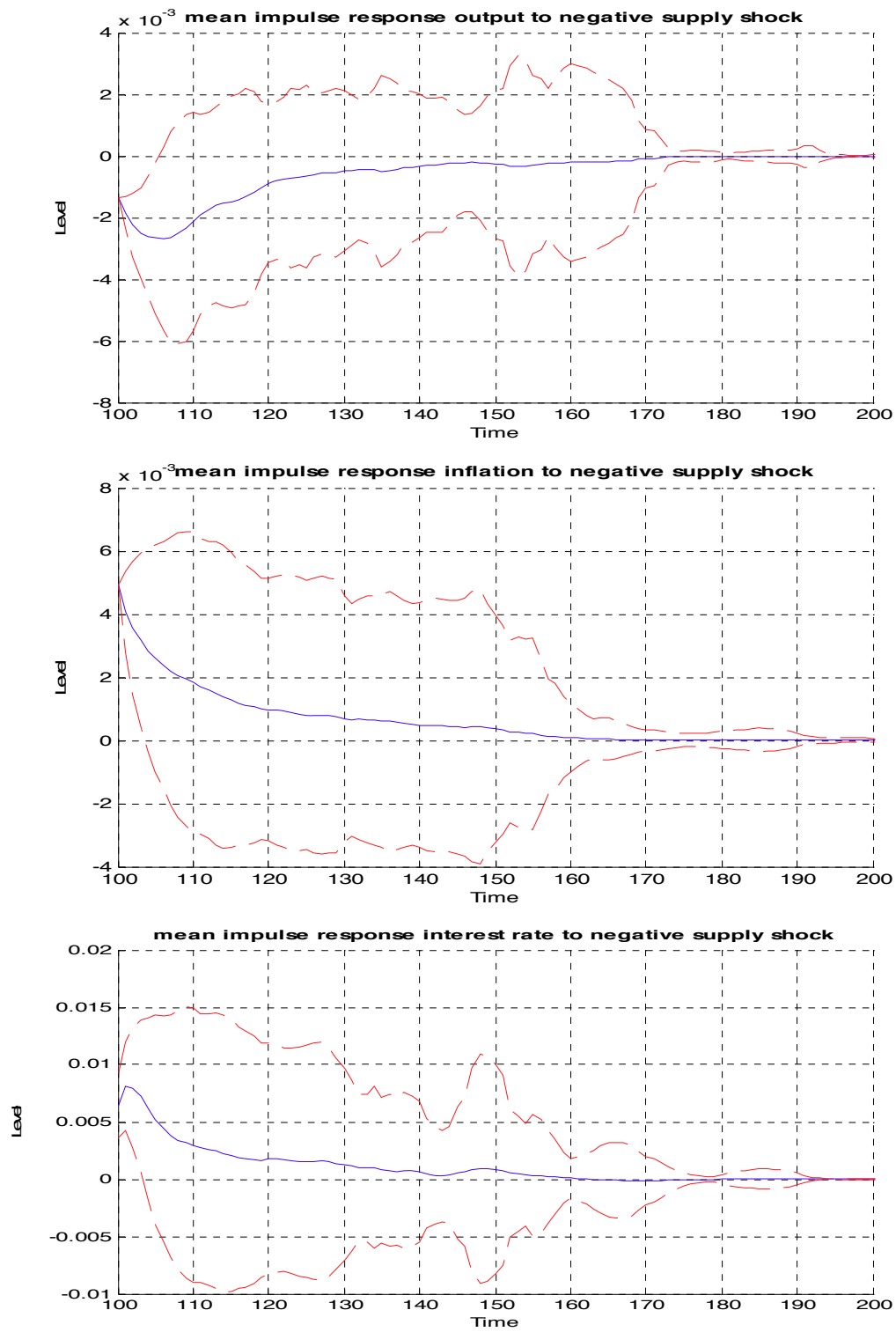
### Rational model

```
pstar = 0;           % the central bank's inflation target
a1 = 0.5;           %coefficient of expected output in output equation
a2 = -0.2;         %a is the interest elasticity of output demand
b1 = 0.5;           %b1 is coefficient of expected inflation in inflation equation
b2 = 0.05;         %b2 is coefficient of output in inflation equation
c1 = 1.5;           %c1 is coefficient of inflation in Taylor equation
c2 = 0.5;           %c2 is coefficient of output in Taylor equation
c3 = 0.5;           %interest smoothing parameter in Taylor equation
sigma1 = 0.005;    %standard deviation shocks output
sigma2 = 0.005;    %standard deviation shocks inflation
sigma3 = 0.005;    %standard deviation shocks Taylor
```

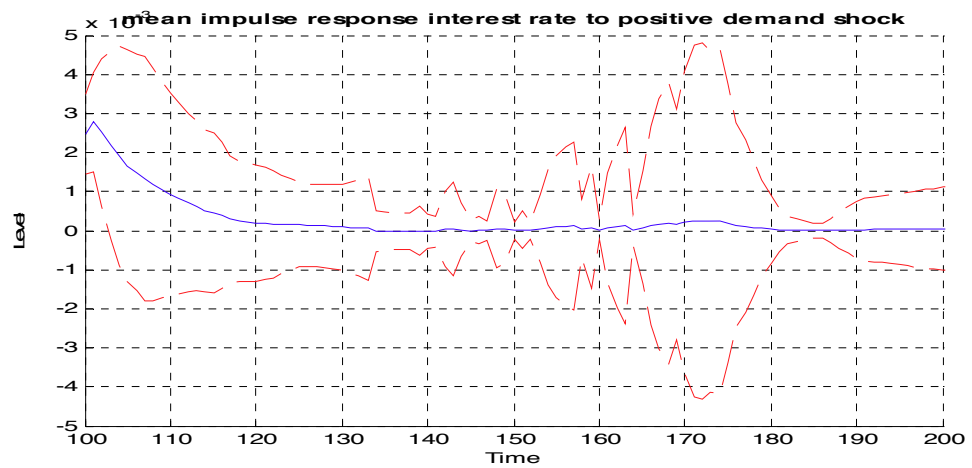
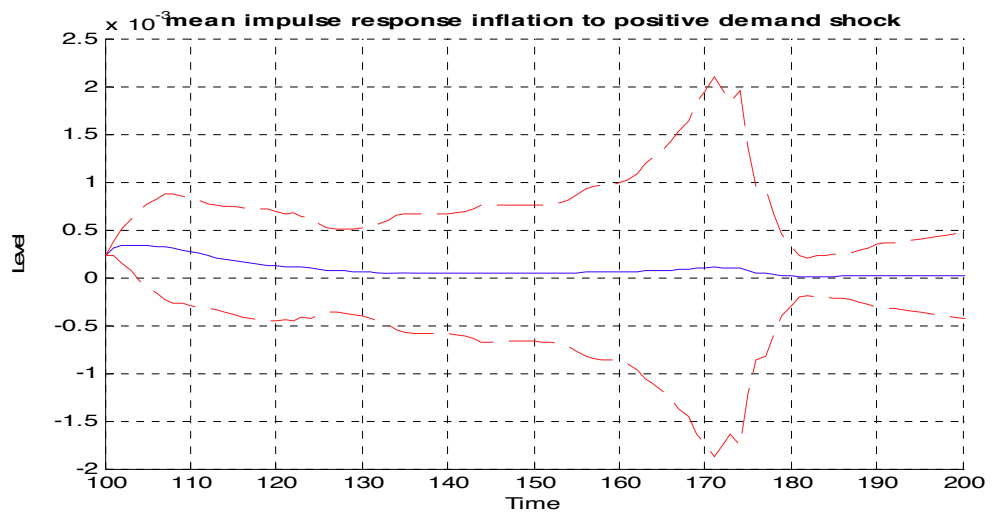
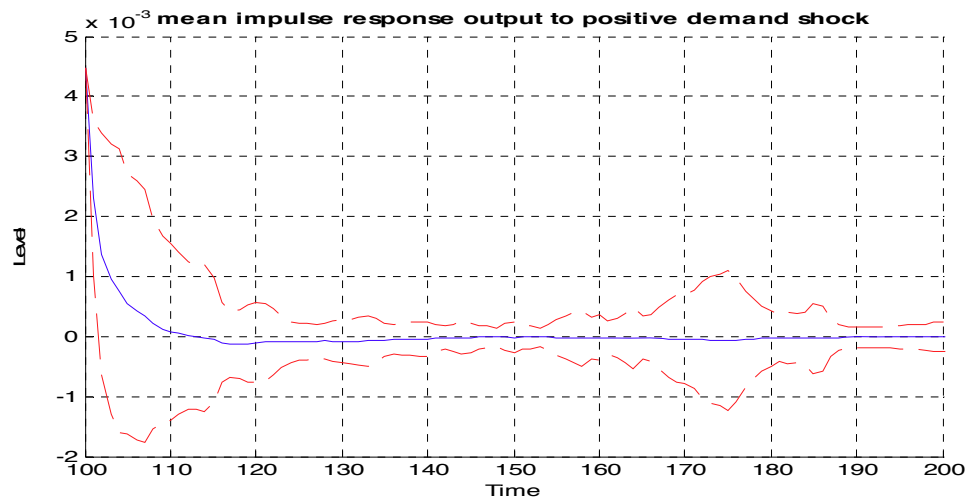
## Appendix B:



## Appendix C: Mean impulse responses to negative supply shock



## Appendix D: Mean impulse responses to positive demand shock



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